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SYNOVIAL FLUID THIXOTROPY IN SQUEEZED-FILM LUBRICATION OF THE HUMAN SYNOVIAL JOINT

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Summary: *The thixotropic (shear-thinning) effect of the synovial fluid in squeeze-film lubrication of the human hip joint is evaluated in the paper. Filtration of the synovial film by articular cartilage is taken into account. Due to a high viscosity of the normal synovial fluid at low shear rates, the squeezed synovial film is found much thicker in a small central part of the lubricated contact at a fixed time after the application of a steady load. Its minimum thickness differs little from the case of the Newtonian fluid with the same high-shear-rate viscosity. The analysis shows that thixotropy of the normal synovial fluid (and the more so that of the inflammatory synovial fluid) is for squeeze-film lubrication of synovial joints irrelevant.*

1. INTRODUCTION

Synovial fluid (SF) forms an interface with the articular cartilage in the synovial joints and serves as a lubricant. It contains a small amount of a macromolecular complex formed by hyaluronic acid and protein. The complex endows the fluid with interesting viscous properties, such as thixotropy. Viscosity η of the normal SF (NSF) increases dramatically with the shear rate γ decreasing below $10^4 s^{-1}$ (Fig. 1). This non-Newtonian effect is low in the inflammatory SF (ISF), differing little from the Newtonian case. It could be speculated that thixotropy of the NSF might be significant for maintaining a continuous fluid film of a sufficient thickness between the articular surfaces for shear rates encountered in the human joints of the lower limb in slow walking or during standing. It has been recently found [1] for the human ankle joint that for the steady surface velocity corresponding to walking, thixotropy of the NSF is not pronounced, as the shear rates in the synovial film are too high.

During standing, the articular surfaces in the human hip joint approach. For axially symmetric articular surfaces and steady axial loading by the body weight (Fig. 2), the shear rates near the centre of the lubricated contact are low and SF thixotropy might apply. The purpose of this paper is to find out whether thixotropy is relevant at least during standing. In the paper, calculations have been made for this configuration, considering articular cartilage a biphasic mixture of the ideal interstitial fluid and a porous elastic, homogeneous and isotropic matrix. This model of articular cartilage [2] (frequently used in the literature) takes into account the fluid transport within the bulk cartilage and across the articular surface, thus

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enabling to describe SF filtration by cartilage due to the fluid pressure gradient in the loaded joint.

2. FORMULATION OF THE PROBLEM

The above-mentioned problem (Fig. 2) leads to the solution of the following partial differential equation (a generalised Reynolds equation)

$$H_{,t}(r,t) = -[rQ(r,t)]_{,r} / r + \hat{q}(r,t) \,.$$

Here r is the radial co-ordinate and t is time. H is the half of the SF film thickness. Q stands for the fluid flow rate across the half of the film thickness H in the r-direction. $\hat{q}$ is the flow rate across the unit articular surface, positive for exudation. In Fig. 1 the experimental points are approximated by the constitutive laws $\eta = m_1$, $\eta(\gamma) = m\gamma^{n-1}$ and $\eta = m_2$ for $\gamma > \gamma_1$, $\gamma_2 < \gamma \le \gamma_1$ and $0 < \gamma \le \gamma_2$, respectively. $m_1, m_2, m, n, \gamma_1, \gamma_2$ are material constants of the SF with $n = 1 + \log(m_1/m_2)/\log(\gamma_1/\gamma_2)$. See the caption of Fig. 3 for their values. It holds that

$$Q = \mp \frac{\gamma_1}{3z_1} H^3 \pm \frac{1-n}{3(2n+1)} (\gamma_1 z_1^2 - \gamma_2 z_2^2) \qquad \text{for } z_1 < H \,,$$

$$Q = \mp \frac{n\gamma_2}{(2n+1) z_2^{1/n}} H^{2+2/n} \mp \frac{1-n}{3(2n+1)} \gamma_2 z_2^2 \qquad \text{for } z_2 < H \le z_1 \,,$$

$$Q = \mp \frac{\gamma_2}{3z_2} H^3 \quad \text{for } z_2 \ge H \,.$$

z_1, z_2 are defined by $z_1 = m\gamma_1 / |p_{,r}|$, $z_2 = m\gamma_2 / |p_{,r}|$. p stands for the SF pressure. For $p(r), \hat{q}(r,t)$ the following formulae have been deduced [3], [6]

$$p(r) = \frac{\rho^4}{2^6 \alpha R} \left[4g \left(1 - \frac{r^2}{\rho^2} \right) - \left(1 - \frac{r^4}{\rho^4} \right) \right], \quad \hat{q}(r,t) = -\beta (r\, p_{,r})_{,r} t^{-1/2} / r \,,$$

$$g = \frac{2^5 \alpha P R}{\pi \rho^6} + \frac{1}{3}, \quad \alpha = \frac{\hat{h}^3}{3\hat{\mu}}, \quad \beta = \hat{h}^2 \left(\frac{\hat{k}}{\pi(\hat{\lambda} + 2\hat{\mu})} \right)^{1/2} .$$

Here P, $\hat{h}$, $\hat{k}$, $\hat{\lambda}$ and $\hat{\mu}$ are the total load, cartilage thickness, cartilage matrix permeability and two elastic Lame's constants of the isotropic cartilage matrix, respectively. R is the effective curvature radius in the sphere-on plane configuration given by $1/R = 1/R_f + 1/R_a$ where R_f, R_a are the radii of the femoral head and of the acetabulum. $R = \hat{R}/2$ for our symmetric case (Fig. 2). The upper (lower) signs in the expressions for Q are valid with $p_{,r} \ge 0$ ($p_{,r} \le 0$). ρ is the radius of the frictionless contact, for which we take the approximate value of the dry contact of two rigid spheres coated by thin elastic isotropic and incompressible layers that is due to Matthewson [4] and is calculated as the real root of the equation

$$g = \frac{1}{2} + \frac{4}{y^2} + \frac{K_1(y)}{2K_1(y) + yK_0(y)}, \quad y = \left(\frac{2}{3} \right)^{1/2} \frac{\rho}{\hat{h}}$$

where K_0, K_1 are the modified Bessel functions.

The above problem has been solved using the method by Hooke [5]. The initial value of H has been taken homogeneous with regard to r and sufficiently large [6]. The following set of geometric, material and loading parameters, considered typical of the human hip joint, loaded by the steady body weight, is used: $\hat{h} = 2\text{mm}, R = 0.5\text{m}, P = 1500\text{N}$, $\hat{\mu} = 0.25\text{MPa}$, $\hat{\lambda} + 2\hat{\mu} = 0.5\text{MPa}$, $\hat{k} = 2\times 10^{-15}\text{m}^4\text{N}^{-1}\text{s}^{-1}$.

3. NUMERICAL RESULTS

Figs. 4 and 5 show the synovial fluid profiles $h(r,t) = 2H(r,t)$ at a fixed t, with and without filtration, respectively. The full, broken and dotted lines refer to the NSF, ISF and the Newtonian SF with constant viscosity m_1, respectively. The film profiles for the non-Newtonian SF form a hump at the contact centre, higher for the NSF. It is apparent (Fig. 4) that homogeneous cartilage filtrates the synovial film intensively, turning SF quickly into a stable synovial gel. Only in a small central part, and for $R > 0$ also in a marginal part of the contact, synovial film remains fluid-like for longer time. Hypothetically, if filtration were absent (Fig. 5), a continuous fluid film would maintain for long. Fig. 5 shows for $\hat{k} = 0$ that on starting at $r = 0$, the difference of h at a fixed t for the two non-Newtonian SFs and the Newtonian fluid decreases with the increasing r, until both non-Newtonian curves merge with the Newtonian one. As the average shear rate in the film at a fixed r decreases with time, the width of the hump increases and its height decreases with time. Interestingly, the minimum film thicknesses at a fixed t of all three SFs are for zero filtration practically the same until about $t = 2\text{s}$. Later, this minimum film thickness becomes higher for the NSF than for the Newtonian SF. However, due to SF filtration by articular cartilage, by that time the fluid film has already turned into a stable gel film (Fig. 4).

4. CONCLUSION

The results received for a homogeneous, isotropic cartilage matrix (modeling the case of early arthritis, with the distinct superficial zone already disrupted or rubbed off) indicate that, at the beginning of the normal approach of the articular surfaces in the loaded hip joint, the shear rate in the synovial film (except near the contact centre) is such high that low value of viscosity, corresponding to high shear rates, applies. The film profiles for the NSF, ISF and Newtonian SF practically merge with the exception of a small vicinity of the contact centre where the NSF realises the highest SF film thickness. Under the step load of 1500N, due to intensive filtration of the synovial film, the squeezed synovial film turns into a stable gel film in the majority of the contact within about 2s (except for small central and possibly also marginal parts). Negative R (with the joint socket deeper than its head) and NSF realises the largest central parts where the SF remains for a long time. Even in the fictitious case of zero filtration, it is as late as some tens of seconds after load application that the shear rates over the whole contact will become such low that the viscosity becomes higher, increasing considerably the minimum synovial film thickness, of order $0.3\mu\text{m}$ at that time. However, due to SF filtration by homogeneous cartilage, by that time the fluid film would have already turned into a stable gel film. Thus, for the homogeneous model of the cartilage matrix the thixotropic effect of the SF in the human hip joint in standing becomes irrelevant.

Transversely isotropic, homogeneous models for the cartilage matrix that simulate different properties of the matrix in tension (that are due to the collagen fibrils) and in compression (that are due to proteoglycans) agree more closely with observed sites of cartilage failure [7]. These models predict not only high shear stress at the subchondral bone interface and separation of the cartilage layer, but are also consistent with lesions observed on the articular surface and provide better curvefits of cartilage early response indentation data

[8]. The mechanical properties of the matrix (such as tension and compression stiffness, permeability) are depth-dependent [9-10]. Especially, the intact superficial zone of the normal articular cartilage (10 to 20% of the total cartilage thickness) has elastic moduli and permeability considerably different (i. e. higher stiffness in tension along the surface, lower stiffness in compression in the direction perpendicular to the surface, lower permeability), as compared to those in the deep zone. A decrease in the tensile stiffness, as well as disruption and disorganization of the collagen network of the superficial zone, have been reported in experimentally induced arthritis in the dog by transection of the anterior cruciate ligament of the knee [11-12]. From this point of view, the models with a homogeneous matrix, where the articular surface is equally compliant and permeable as the bulk material, may actually describe articular cartilage with the superficial collagen network already disrupted and degraded, the case of early osteoarthritis. Quite recently, the case of a biphasic, bilayered, transversely isotropic articular cartilage, modeling the normal cartilage with the intact superficial zone, has been also accomplished [13]. The articular surface deflection, fluid flux and SF pressure distribution have been obtained. The procedure is the same as in the homogeneous, isotropic case, but the expressions for $p(r), \hat{q}(r,t)$ and ρ are more complicated. For this case, filtration is lower and the SF film maintains some seconds longer compared with the homogeneous case that models early osteoarthritis, with the surface zone already disrupted or worn away. Thus, SF thixotropy also remains irrelevant for the normal articular cartilage with the intact superficial zone (modeled as bilayered), as it is an intermediate case to the homogeneous, porous case and to the non-porous case.

Finally, it must be recalled that many simplifying assumptions have been made in the mathematical modeling of the problem in question, in order to receive an analytic solution. These are: axial symmetry; linearly elastic, intrinsically incompressible, porous cartilage matrix; ideal interstitial fluid; impermeable, rigid subchondral bone and small deformation of the matrix. Mechanical behaviour of the cartilage matrix near the free surface seems to control SF filtration. Unfortunately, there is still much uncertainty in the mechanical properties of the cartilage matrix near the articular surface. To date, moreover, there is no experimental verification of SF filtration supporting the theoretical results received.

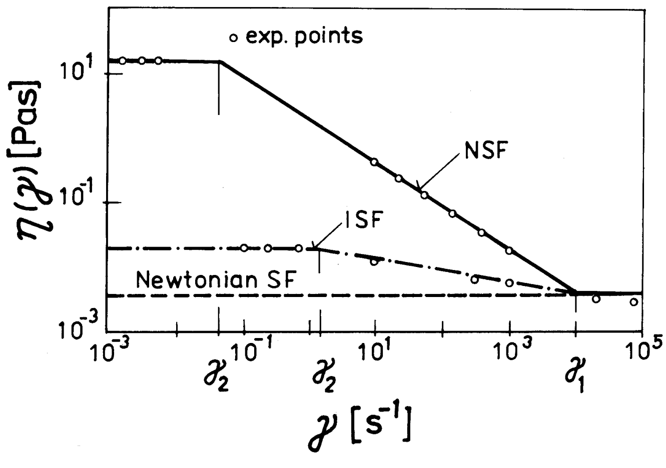

Fig. 1 Viscosity $\eta(\gamma)$ of NSF and ISF taken from the human knee joints (measured points are taken from [14])

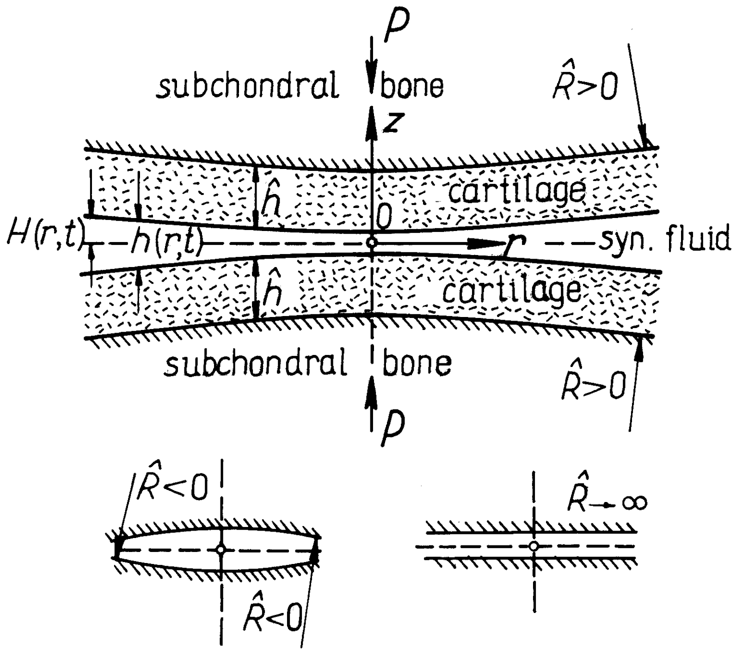

Fig. 2 Lubricated contact: the axis of symmetry: $r = 0$, the plane of symmetry: $z = 0$

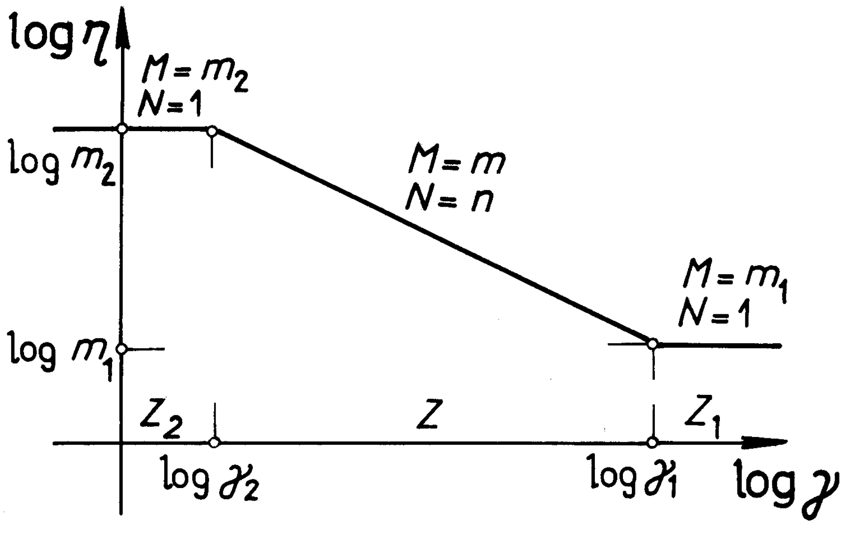

Fig. 3 Viscosity parameters: NSF: $m_1 = 4\times10^{-3}\,\mathrm{Pas}$, $m = 1.99\,\mathrm{Pas}^n$, $m_2 = 15\,\mathrm{Pas}$, $n = 0.326$, $\gamma_1 = 1\times10^4\,\mathrm{s}^{-1}$, $\gamma_2 = 5\times10^{-2}\,\mathrm{s}^{-1}$; ISF: $m_1 = 4\times10^{-3}\,\mathrm{Pas}$, $m = 2.3\times10^{-2}\,\mathrm{Pas}^n$, $m_2 = 2\times10^{-2}\,\mathrm{Pas}$, $n = 0.811$, $\gamma_1 = 1\times10^4\,\mathrm{s}^{-1}$, $\gamma_2 = 2\,\mathrm{s}^{-1}$

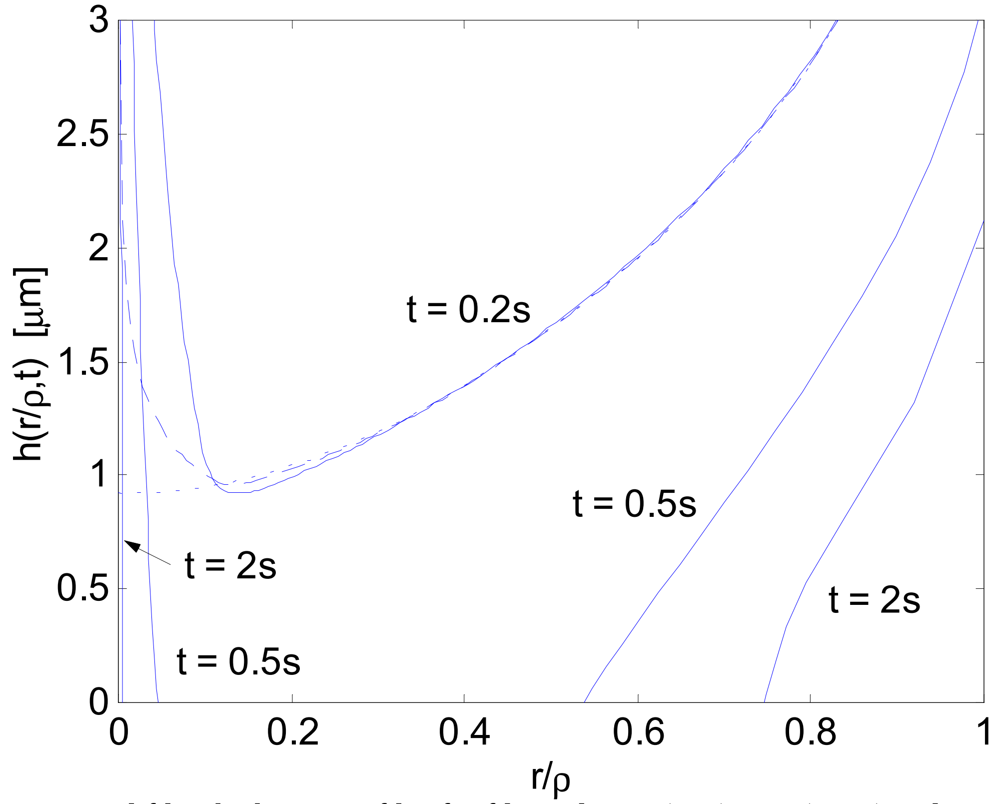

Fig. 4 Synovial film thickness profiles for filtrated NSF (___), ISF (_ _ _) and Newtonian SF (…..) of the same high-shear-rate viscosity

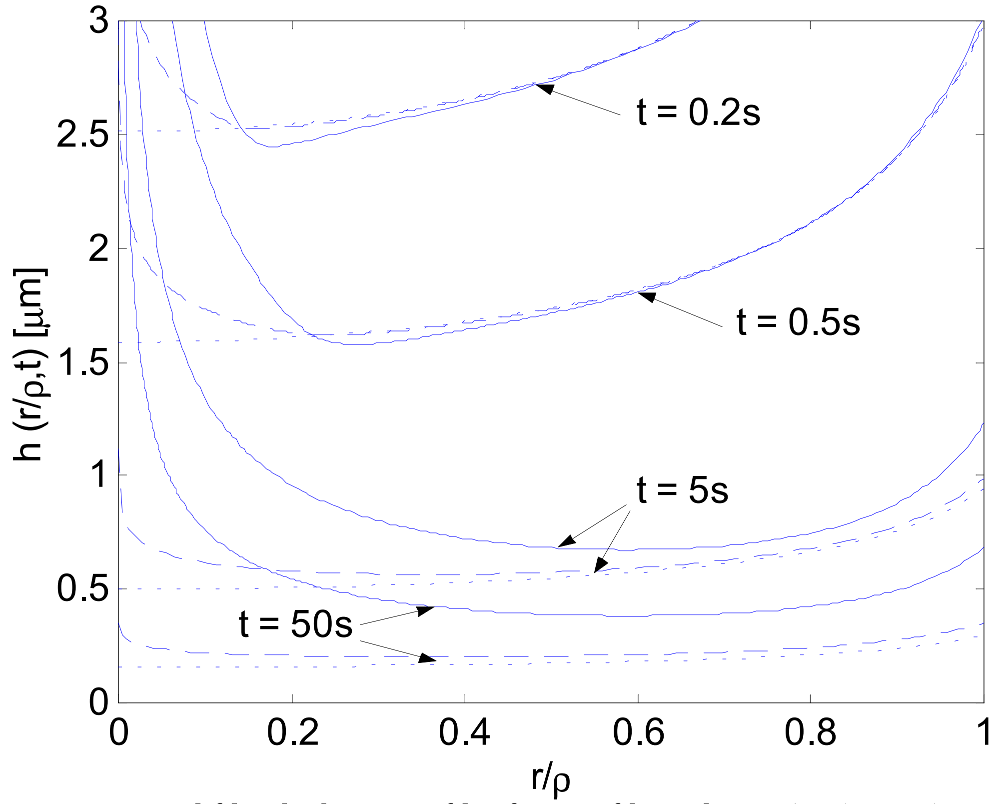

Fig. 5 Synovial film thickness profiles for non-filtrated NSF (___), ISF (_ _ _) and Newtonian SF (…..) of the same high-shear-rate viscosity

5. Acknowledgement

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6. References

1. Hlaváček M.: A central film thickness formula for elastohydrodynamic lubrication of cylinders with soft incompressible coatings and a non-Newtonian piecewise power-law lubricant in steady rolling motion, Wear 205 (1997), 20-27.
2. Mow V. C., Kuei S. C., Lai W. M., Armstrong C. G.: Biphasic creep and stress relaxation of articular cartilage: theory and experiment, Trans. ASME, Journal of Biomechanical Engineering 102 (1980), 73-84.
3. Hlaváček M.: The role of synovial fluid filtration by cartilage in lubrication of synovial joints—II. Squeeze-film lubrication: homogeneous filtration, Journal of Biomechanics 26 (1993), 1151-1160.
4. Matthewson M. J.: Axi-symmetric contact on thin compliant coatings, Journal of Mechanics and Physics of Solids 29 (1981), 89-113.
5. Hooke C. J.: The elastohydrodynamic lubrication of a cylinder on an elastomeric layer, Wear 111 (1985), 83-99.
6. Hlaváček M.: Squeeze-film lubrication of the human ankle joint with synovial fluid filtrated by articular cartilage with the superficial zone worn out, Journal of Biomechanics 33 (2000), 1415-1422.
7. Donzelli P. S., Spilker R. L., Ateshian G. A., Mow V. C.: Contact analysis of biphasic transversely isotropic cartilage layers and correlations with tissue failure, Journal of Biomechanics 32 (1999), 1034-1047.
8. Cohen B., Gardner T. R., Ateshian G. A.: The influence of transverse isotropy on cartilage indentation behavior—A study of the human humoral head, Transactions Orthopaedic Research Society, Orthopaedic Research Society, Chicago, p. 185.
9. Akizuki S., Mow V. C., Muller F., Pita J. C., Howell D. S., Manicourt D. H.: Tensile properties of the knee joint cartilage: I. Influence of ionic condition, weight bearing and fibrillation on the tensile modulus, Journal of Orthopedic Research 4 (1986), 379-392.
10. Schinagl R. M., Gurskis D., Sah R. L.: Depth-dependent confined compression modulus of full-thickness bovine AC, Journal of Orthopaedic Research 15 (1997), 499-506.
11. Guilak F., Ratcliffe A., Lane N., Rosenwasser M. P.: Mechanical and biochemical changes in the superficial zone of articular cartilage in canine experimental osteoarthritis, Journal of Orthopaedic Research 12 (1994), 474-484.
12. Stockwell R. A., Billingham M. E. J., Muir H.: Ultrastructural changes in articular cartilage after experimental section of the anterior cruciate ligament of the dog knee, Journal of Anatomy 136 (1983), 425-439.
13. Hlaváček M.: The influence of the acetabular labrum seal, intact articular superficial zone and synovial fluid thixotropy on squeeze-film lubrication of the spherical synovial joint (submitted).
14. Zeidler H., Altman S., John B., Gaffga R., Kulike W. M.: Rheologie pathologischer Gelenkflussigkeiten—I. Weitere Ergebnisse zur Viscoelastizitat, Rheologica Acta 18 (1979), 151-167.