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VIBRATIONS OF ROTOR SUPPORTED IN MAGNETIC BEARINGS WITH IMPACTS

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Abstract:

The impact motion of rigid rotor supported on passive magnetic bearings is studied. Impacts on passive magnetic bearings are investigated. Impacts occur in retainer bearing, which is usually a ball or roller bearings at magnetic supports. The forces arising during oblique impacts are modelled by dynamical Hertz's contact including material damping, Coulomb dry friction and by viscous damping proportional to the tangential velocity.

The influences of various values of Hertz's stiffness, combined with three kinds of damping are studied by numerical simulation. The results are presented in the form of response curves and trajectories of motion in X, Y plane. Various kinds of impact motion – periodic, with single or multiple periods, quasi-periodic and chaotic oscillations were found. The frequency intervals and transitions between different kinds of motions were recorded.

Key words: Passive magnetic bearings, impacts in retainer bearing, response curves, plane trajectories.

1. Introduction

Magnetic bearings are progressive elements in the mechanical engineering, because they have a lot of advantages in comparison with the other conventional types of bearings, e.g. fluid-film bearings, oil-less, ball or roller bearings, etc. The most important advantage of magnetic bearings is their zero drag against rotation, but they have also a lot of other merits. There are two basic principles of magnetic levitation:

a) Active bearings (AMB) using electromagnets fed by currents, which must be controlled by means of feedback loops in order to stabilize the rotor in the given position. The control device enables to tailor the stiffness and radial damping according to operating conditions.

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b) Passive magnetic bearings (PMB), using strong permanent magnets, are more simple then AMB, as they do not need any feedback loop and no external input of energy, but they have fixed properties, which cannot be controlled during operation. They have also very low radial damping, so they are more inclined to unstable motions.

Both active and passive magnetic bearings are very suitable for the strong operating conditions e.g. for very low temperature, radiation, operation in vacuum etc. All types of magnetic bearings must for safety reasons contain central safety components – emergency retainer bearings with sufficient clearance between shaft and stator. They protect magnetic bearings from direct contact with rotor and retain the amplitudes of vibrations in safe limits after their undesirable increase.

In this paper we investigate the impact motion of magnetically supported rigid rotor in the case of contact with retainer bearing. There are a lot of studies concerning to the problem of rotor/stator interaction with impacts and rubs [1-5] but it is usually supposed that the rotor impacts on non-rotating surface of fixed hole in stator. The retainer bearing is usually a ball or roller bearing, whose inner ring sets into rotation after oblique impacts. The combination of dry Coulomb friction with viscous and material damping is used to describe this behaviour. Radial stiffness is given largely by contact deformation between shaft and inner ring, inner ring and ball and between ball and outer ring. Therefore the contact Hertz's law is used.

The PMB have usually lower radial magnetic stiffness than AMB and therefore we shall focus here on the properties of PMB. The difference between vertical and horizontal rotor is considered with respect to the influence of weight: $mg = 0$ for vertical shaft, $mg \neq 0$ for horizontal shaft.

2. Passive magnetic bearing assembly

Draft of the passive magnetic bearing used in experimental rotor set-up in Institute of Thermomechanics AS CR, Prague, is in Fig. 1a. Permanent magnetic elements are in the form of circular rings of diameter D=60 with axial magnetic orientation between two neighbouring rings. Cap d=1.0 mm is constant during the relative motion of rings against each other. Radial displacement x causes the deformation of magnetic lines and produces the returning force F. A retainer bearing 2 with clearance r_h= 0.5 mm limits the amplitudes of shaft 3 (R_1=14 mm) [6,7].

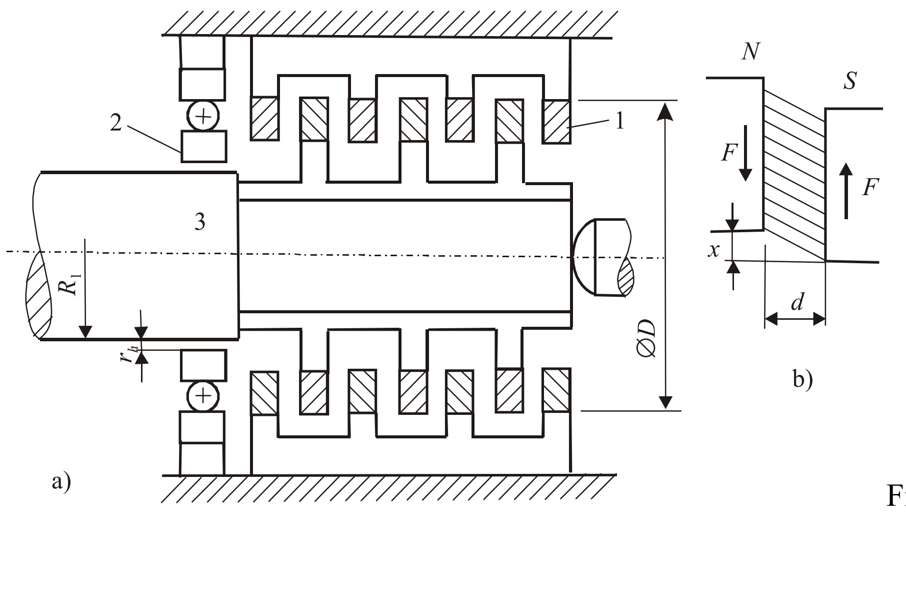

Fig. 1

Some rotors (e.g. centrifuges) have the centre of mass very close to the one of the bearing. The motion can be then investigated in one plane *x*, *y*. A planar model can be used also at assumption that a cylindrical motion of rotor supported on two bearings exists [8]. In this contribution, we shall focus our attention to this 2DOF plane problems.

The magnetic central forces for small declination of rotor $0< r <r_h$ are weakly nonlinear with cubic characteristic

$$F_m = kr\left(1+k_2 r^2\right). \tag{1}$$

3. Motion with impacts in retainer bearing

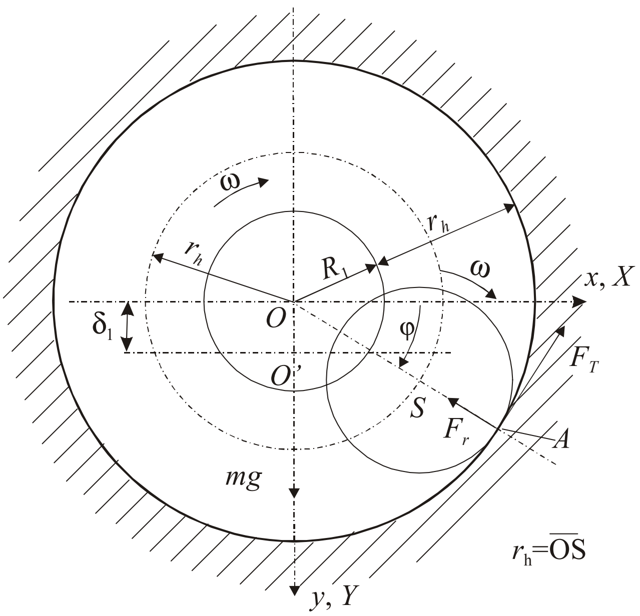

Fig. 2

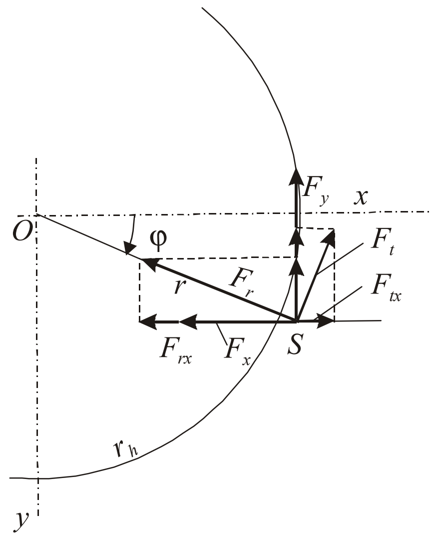

Fig. 3

The radial force F_r is the Hertz's dynamical contact force

$$F_r = k_h\left(r - r_h\right)^{3/2}\left(1 + b_h \dot{r}\right), \tag{2}$$

which has to be added to the central magnetic force (1). Tangential force is composed of Coulomb friction proportional to F_r and viscous resistance F_t which we suppose to be proportional to the tangential velocity v_t of the shaft surface in contact point

$$F_t = f\,\mathrm{sgn}\left(v_t\right)F_r + b_t v_t, \tag{3}$$

where is F_r given by (2),

b_t coefficient of viscous damping,

$v_t = \left(\dot{y}x - \dot{x}y\right)/\sqrt{x^2+y^2} + R_1\omega$ is tangential velocity.

After addition to the magnetic forces and decomposition of these resistance forces into coordinates directions x, y we can include them into equations of planar motion of centre S.

We get

$$\begin{aligned}
&m\ddot{x}+b\dot{x}+kx+k_3x(x^2+y^2)+H(r-r_h)\{\ k_h(r-r_h)^{3/2}(1+b_h(\dot{x}x+\dot{y}y)/r)*\\
&[x/r-y/r\,f\,\mathrm{sgn}((\dot{y}x-\dot{x}y)/r+R_1\omega)]-y/r\,b_t((\dot{y}x-\dot{x}y)/r+R_1\omega)\ \}=me\omega^2\cos\omega t,\\
&m\ddot{y}+b\dot{y}+ky+k_3y(x^2+y^2)+H(r-r_h)\{k_h(r-r_h)^{3/2}(1+b_h(\dot{x}x+\dot{y}y)/r)*\\
&[y/r+x/rf\,\mathrm{sgn}((\dot{y}x-\dot{x}y)/r+R_1\omega)]+x/r\,b_t((\dot{y}x-\dot{x}y)/r+R_1\omega)\ \}=me\omega^2\sin\omega t+mg.
\end{aligned} \tag{4}$$

where $k_3=k*k_2$, e is eccentricity of rotor, H is Heaviside function enables to describe motion with and without impacts by means of one set of equations.

For the numerical simulation, it is convenient to transform equations (4) into dimensionless form:

$$\begin{aligned}
&X''+BX'+X+\kappa_3X(X^2+Y^2)+H(R-1)\{\kappa_h(R-1)^{3/2}(1+B_h(X'X+Y'Y)/R)*\\
&[X/R-Y/Rf\,\mathrm{sgn}((Y'X-X'Y)/R+R_r\eta)]-Y/R\,B_T((Y'X-X'Y)/R+R_r\eta)\ \}=Ec\eta^2\cos\eta\tau\\
&Y''+BY'+Y+\kappa_3Y(X^2+Y^2)+H(R-1)\{\kappa_h(R-1)^{3/2}(1+B_h(X'X+Y'Y)/R)*\\
&[Y/R+X/Rf\,\mathrm{sgn}((Y'X-X'Y)/R+R_r\eta)]+X/R\,B_T((Y'X-X'Y)/R+R_r\eta)\ \}=Ec\eta^2\sin\eta\tau+G,
\end{aligned} \tag{5}$$

where

$$X=x/r_h,\quad Y=y/r_h,\quad R=r/r_h=\sqrt{X^2+Y^2},\quad R_r=R_1/r_h,\quad Ec=e/r_h,$$

$$\tau=t\sqrt{k/m},\quad \eta=\omega\sqrt{m/k},\quad B=b/\sqrt{km},\quad \kappa_3=r_h^2k_3/k,\quad \kappa_h=\sqrt{r_h}\,k_h/k, \tag{6}$$

$$B_h=b_h/\sqrt{km},\quad B_T=b_t/\sqrt{km},\quad G=mg/(kr_h).$$

4. Numerical simulation

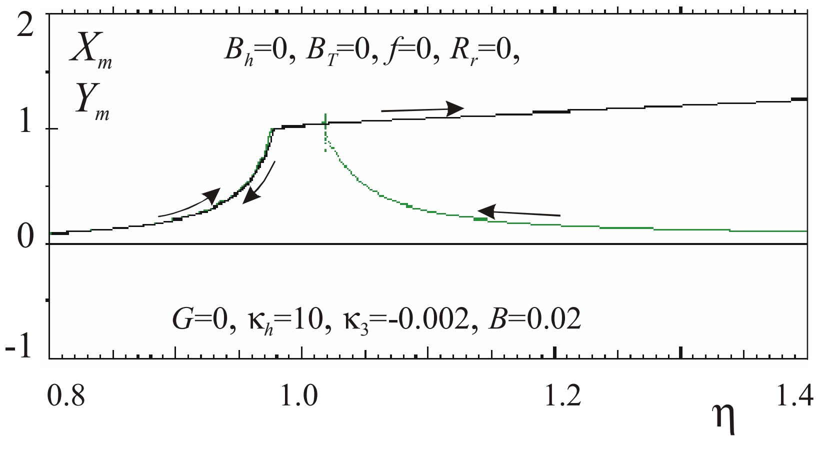

Fig. 4

Quasi-stationary and non-periodic oscillation of shaft motion in retaining bearing was investigated by numerical simulation in Pascal. Equations of motion contain many parameters characterizing the elastic and damping properties of studied vibro-impact system. Here we restrict the presentation to the demonstration of the influence of Hertz's coefficient $k_h(\kappa_h)$, dry friction coefficient f and damping coefficient for radial b_h (B_h) and tangential b_t(B_T) velocity on the course of response curves of impacting mass point m (R_1=0) and on time history and plane trajectories of oscillations at different excitation frequencies. Response curve (X_m,η , Y_m,η) of system with very weak impacts in retainer bearing is shown in Fig. 4 for values κ_h=10, κ_3=-0.002, R_r=0, G=0 and for no damping B_h=0, B_T=0, f=0. The upper, almost horizontal, branch ascertains amplitudes of circulating mass in the nonlinear central symmetric potential field, which is typical for vertical rotors. Due to the symmetry, the response curves X_m,η , Y_m,η are identical. Next Figure 5 presents response curves of system with the same parameters but loaded by the weight G=0.3.

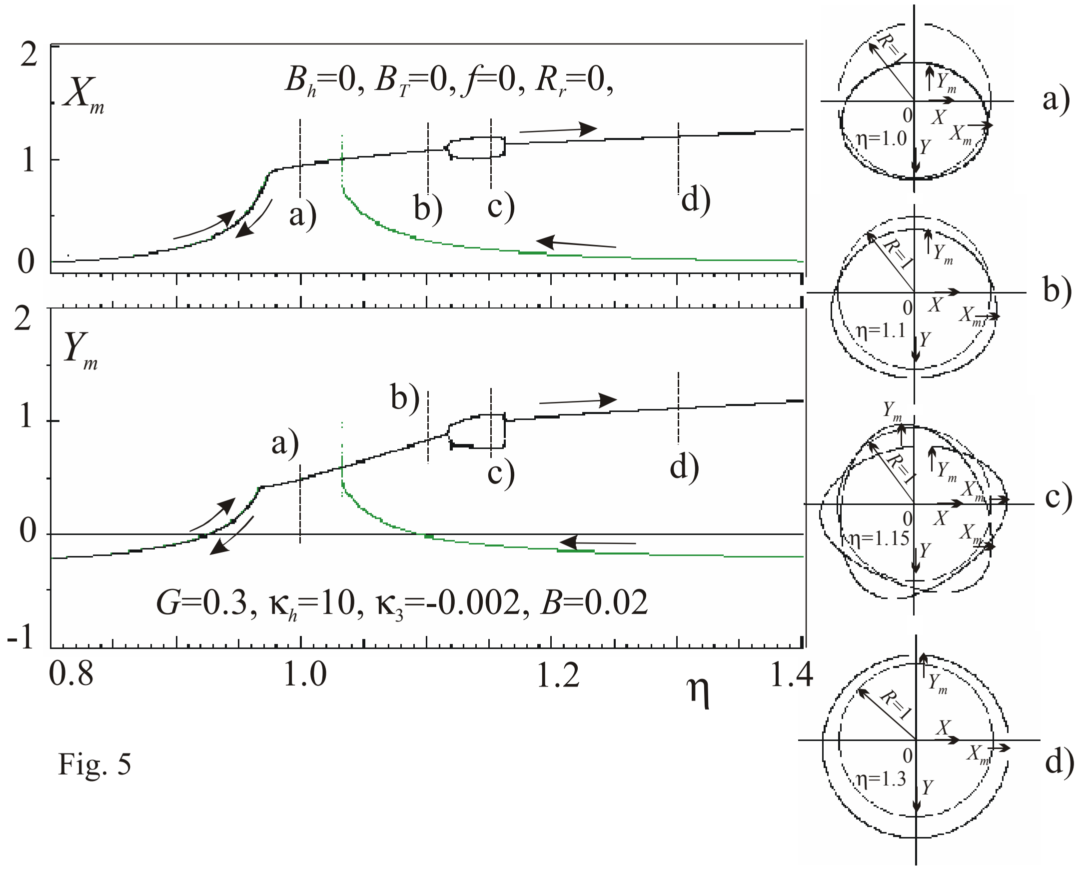

Fig. 5

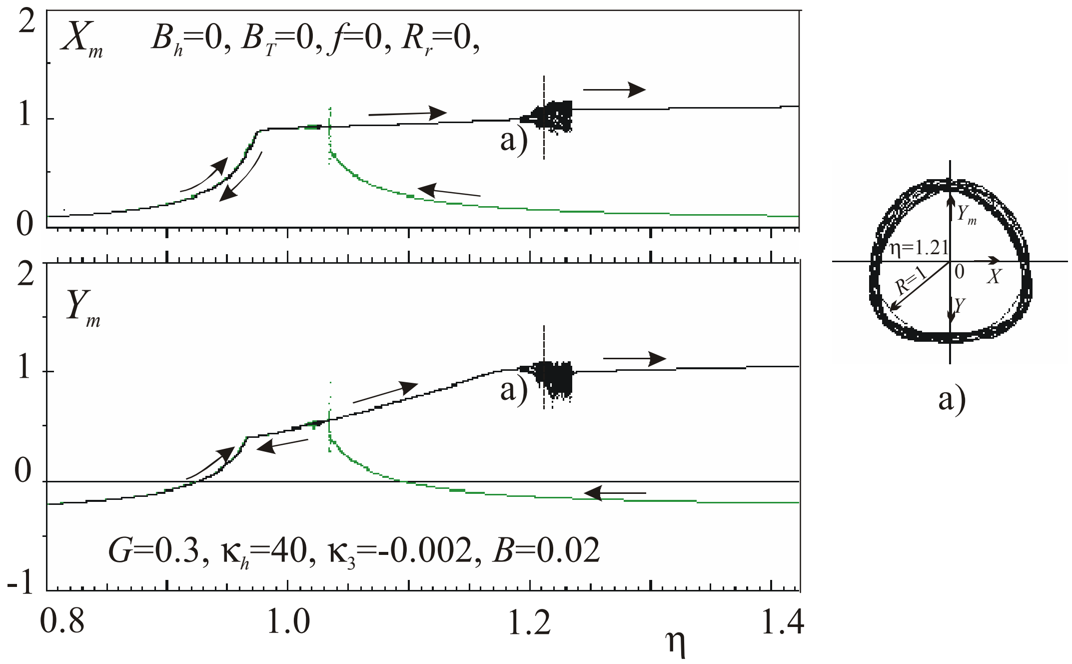

Fig. 6

Response curve in vertical direction Y differs from the horizontal one X. Another differences are: increasing part of response curve in the range $\eta \in (0.97, 1.12)$ and double line near η=1.2. Phase trajectories at η=1; 1.1; 1.15; 1.3 are on the right of Fig. 5. The increase of Hertz's coefficient to κ_h=40 results in extinction of double line and in an existence of a new chaotic motion near η=1.2. Type of chaotic motion is seen from the phase trajectories pasted on the right side of Fig. 6.

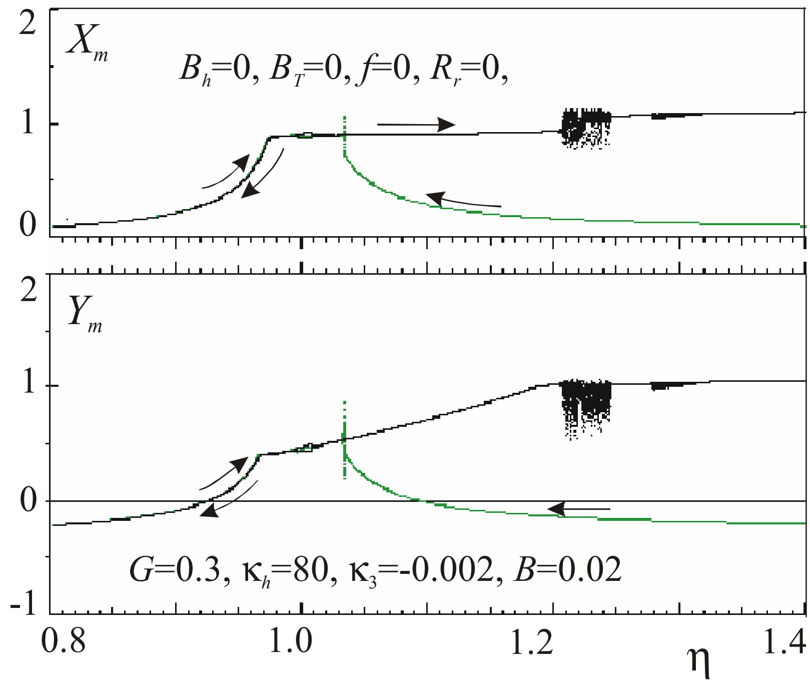

Fig. 7

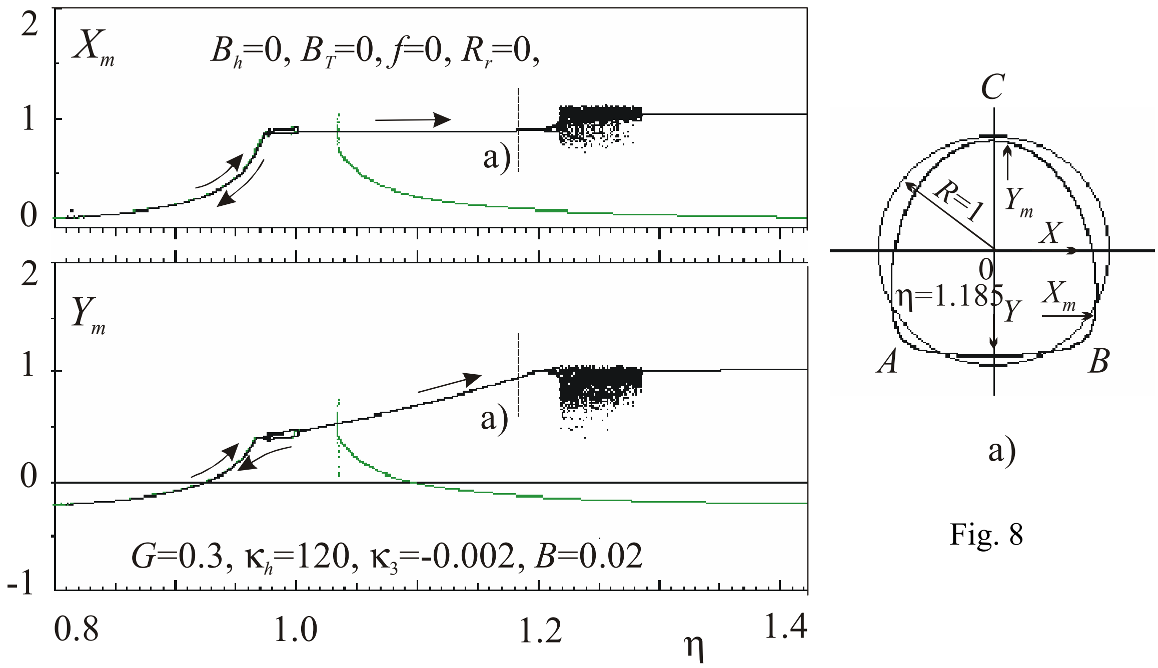

Fig. 8

Twice higher Hertz's stiffness coefficient κ_h=80 does not change considerably the feature of response curves, as it is seen from Fig. 7. Also further rise of κ_h to κ_h=120 (Fig. 8) does not bring any essential changes in response curves. Noteworthy is the course of motion X, Y trajectory after bottom contact with retainer bearing e.g. at η=1.185, where two bottom impacts A, B occur (Fig. 8a). When the rotor begins to touch the retainer bearing in the third point C, the periodic oscillations transfer into chaos in frequency interval $\eta \in (1.21,\ 1.29)$.

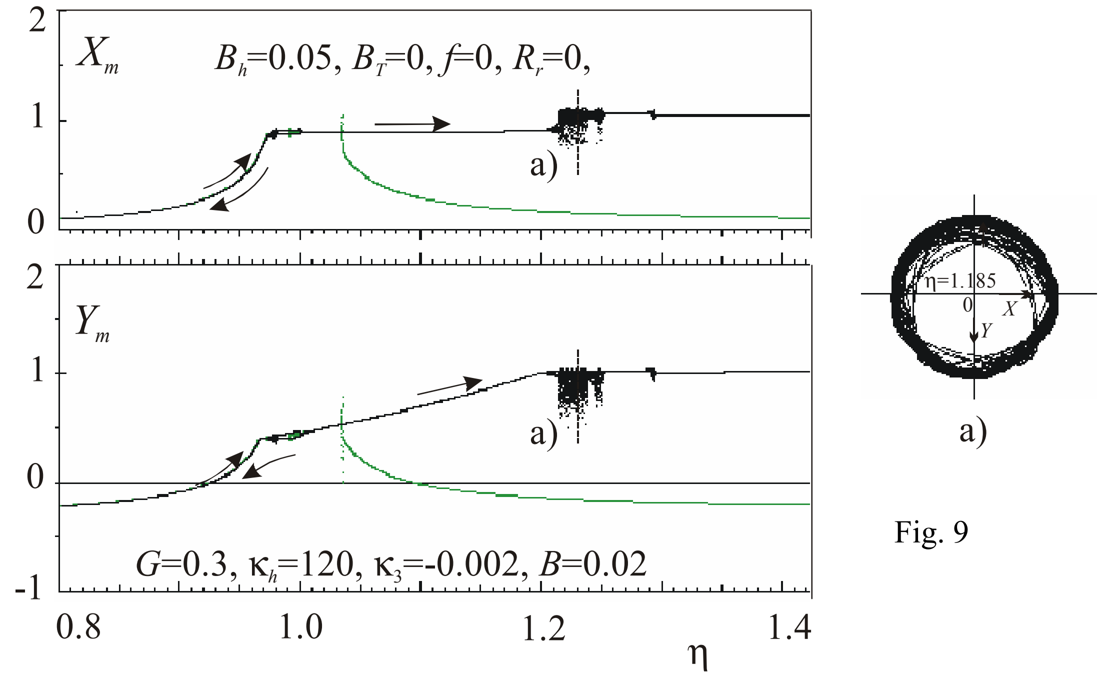

Fig. 9

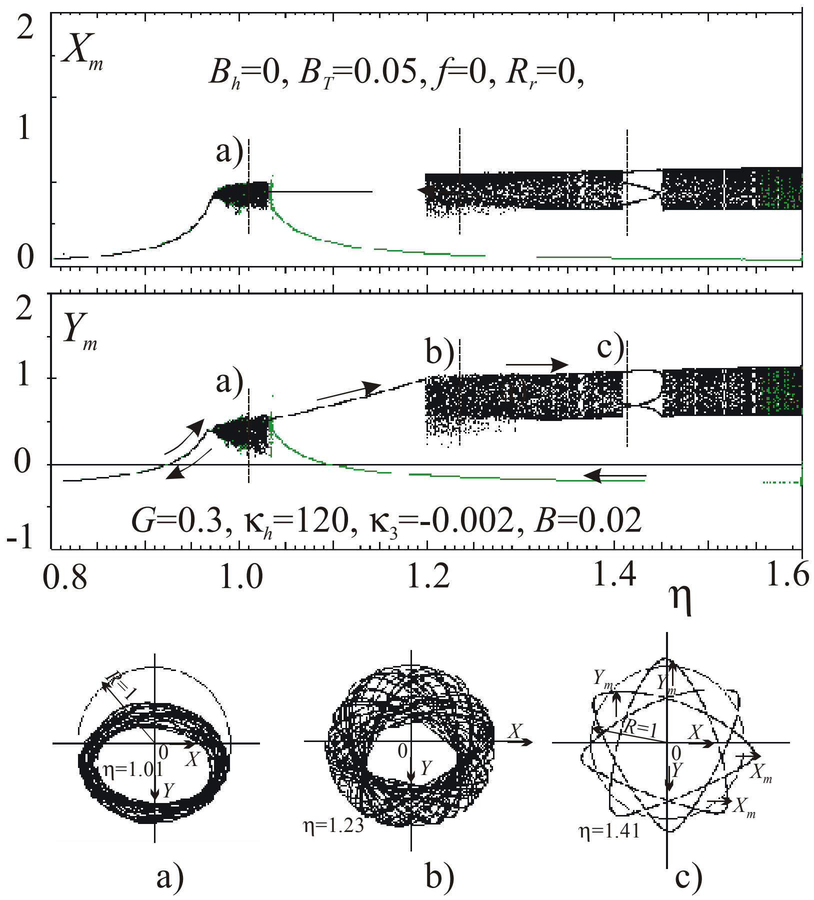

Fig. 10

Let us mention the influences of different kinds of damping. The internal material damping during the impacts is given by the parameter $b_h(B_h)$. Damping component of Hertz's dynamic force is

$$F_d = k_h\left(\sqrt{x^2+y^2}-r_h\right)^{3/2} b_h(\dot{x}x+\dot{y}y)/\sqrt{x^2+y^2}$$

or in dimensionless variables

$$\frac{F_d}{kr_h} = \kappa_h (R-1)^{3/2} B_h\left(\dot{X}X+\dot{Y}Y\right)/R, \quad \text{where } R=\sqrt{X^2+Y^2}\,. \tag{7}$$

This material damping force has the radial direction, without any tangential component and acts only in short time interval during the impact. Its influence on the course of response curve is therefore insignificant (see differences between Fig. 8 and Fig. 9 where B_h=0.05).

Viscous damping characterized by coefficient B_T influences very considerably the impact motion, as it is seen from Fig. 10 for B_T=0.05. The different kinds of motions alternate at increasing frequency η. The impactless motion passes to chaos with impacts in bottom half of retainer bearing (Fig. 10a) changes into periodic oscillations, which at η=1.2 jumps again into chaos (Fig. 10b), but with impacts on the whole surface of retainer bearing. Near η=1.40, the periodic oscillations with $T=3\dfrac{2\pi}{\omega}$ and 8 impacts in period T stabilizes. It turns back into chaotic oscillations at η=1.45 (Fig. 10c).

The dry friction coefficient f influences also the response in a great scale. Corresponding response curves are shown in Fig. 11, where only dry friction with f=0.05 damped the motion. Response is again characterized by alternation of chaotic motion (η=0.97-1.05) with periodic one at η=1.05-1.07 with long period $T=6\dfrac{2\pi}{\omega}$ and 8 impacts only on bottom half of retainer bearings (Fig. 11b). In frequency interval η=1.14-1.19, simple periodic motion exists. It changes into chaos at η=1.19. The X, Y plane trajectories are shown in Fig. 11c. Let us point out that only two small segments of perimeter of retainer bearing are loaded by impacts (Fig. 11c). Similar properties have periodic oscillations ($T=10\dfrac{2\pi}{\omega}$) in interval η=1.3-1.33 (Fig. 11d). Then after narrow interval of chaotic oscillation, the impact motion loses its stability and jumps to the impactless oscillations.

All aforementioned results concern the system with concentrated mass point (R_r=0). The rotor system with shaft radius R_1 ten times greater than the clearance r_h i.e. R_r=10 shows similar properties as Fig. 11. Corresponding graphs will be presented on conference.

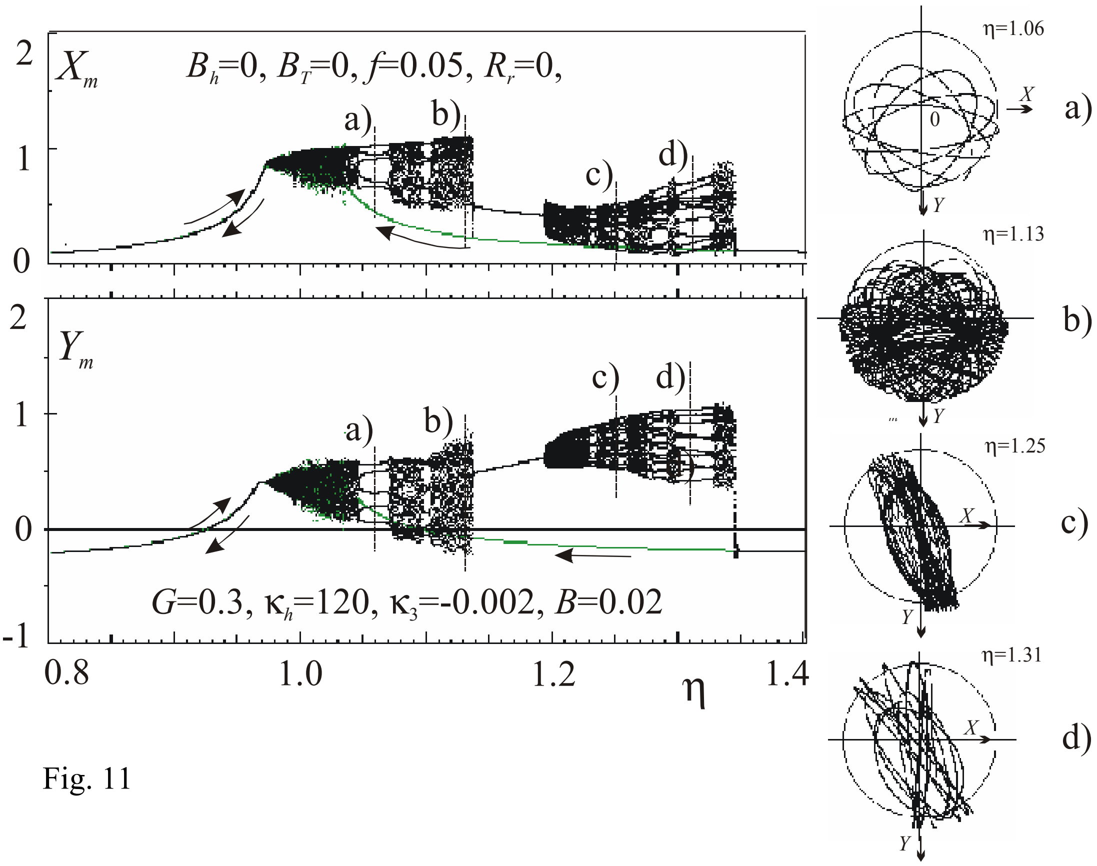

Fig. 11

5. Conclusion

The retainer bearing, used for safety reasons in mechanical engineering for rotors supported on magnetic bearings, is very important element with strongly nonlinear elastic and damping properties. Derived mathematical model of forced motion inside retainer bearing enables to study the influences of main system parameters, as stiffness, material and viscous damping, dry friction etc. on the course of response curves and on the trajectories of journal motion. Various forms of the periodic, quasi-periodic and chaotic motion with impacts were unveiled by means of numerical simulation. It was shown that the dynamical behaviour depends very strongly on the stiffness and damping parameters.

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