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LONGITUDINAL DYNAMICS OF RACE CARS

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Summary: *The article focuses on optimisation of gear ratios of race cars. Whereas gear ratios of production cars are a trade-off between various user requirements and manufacturer's business plan, race car gear ratios are based on a specific assignment to cover given distance (usually one race lap) in a minimum time. In this article, we will describe a procedure of lap time calculation based on data measured and logged during the race or training.*

1. Introduction

Contemporary motor sport is characterized by a tough competition. As such, it often relies upon scientific knowledge and modern technologies that are available not only for F1 teams, but also for racing teams of lower levels. Today, race cars are equipped with systems of collection of data during car operation, i.e. racing dataloggers. Dataloggers usually record the following parameters: rpm of one of the wheels, engine rpm, steering wheel angle, throttle angle, transversal acceleration, cooling liquid temperature, oil temperature and pressure, fuel pressure, battery voltage etc. However, there are also systems that can measure even forty channels. Using the data logged during car operation, we can analyze racer's driving style, car handling in specific track segments, appropriateness of gear ratios etc. Besides datalogger information, motor sport teams use various simulation programs for calculation of driving dynamics.

Lap times of individual drivers differ only slightly. It happens that times of three or five racers differ by less than a hundredth of second. Every slight improvement is therefore very valuable.

Lap times can be possibly reduced by optimum gearing. Today's race cars (formulas) are equipped with transmissions that can change gear wheels for individual gears (including axle drive gear sets) in relatively short time. In fact, team staff can change gear ratios between two trainings of the day.

Assessment of gear ratios is usually done on the basis of information from racing dataloggers. For example, the following parameters are monitored: engine rpm at the end of track turn, maximum engine rpm at the end of straight track segment, or duration of engine rpm being limited by speed limiter. Also, the necessity to change gears in extremely difficult track segments is monitored.

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A higher level of optimisation of gear ratios is represented by Lap Time Simulation Software that can calculate the best possible lap time for a given car and circuit (see Cambiaghi et al., 1996; Candelpergher et al., 2000; Thomas et al., 1996). Calculations of the best lap times are based on various strategies, e.g. direct simulation, reverse simulation etc. Some programs can optimise car velocity on a track and also find the optimum cornering lines. Other applications optimise only dynamics of trajectory derived from datalogger inputs. Obviously, the more sophisticated software, the more time should be reserved for calculations, and the more input data will be needed.

This article describes a simplified assessment of gear ratio selection based on straight track segments times only. This approach makes calculations relatively fast and requires only a limited number of input data.

2. Calculation procedure

The procedure is based on various driving modes of a car on race track. Racers drive their cars with the aim to achieve a maximum total acceleration in each segment of the track. They put the brakes on before a turn, achieve a maximum lateral acceleration in a turn, accelerate at the end of the turn and try to achieve a maximum longitudinal acceleration in subsequent straight track segment. In this way, they try to reach a maximum speed. Afterwards, they accelerate as long as possible until they are forced to slow down not to exceed a maximum attainable lateral acceleration in the next turn. Of course, selection of gears can affect only some driving modes - e.g. speed behaviour of a car that brakes before a turn is not directly influenced by gear ratios. Similarly, behaviour of the car passing through a turn is not directly affected by gear ratios either. On the contrary, acceleration at the end of a turn and subsequent acceleration in straight track segment is fully dependent on selection of gear set in gearbox.

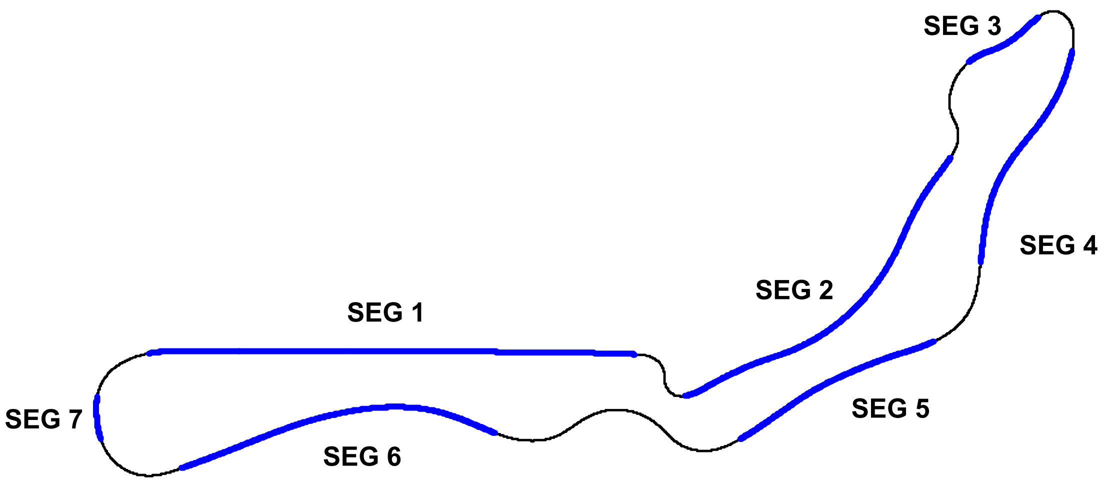


Figure 1: Course map of the Autodrome in Most with marked segments (SEG 1 - SEG 7)

In order to identify the impact of gear ratios on final lap time, it is necessary to pick up appropriate segments of the track. Afterwards, car movements in all these segments should be mathematically simulated so that the time needed to cover an adequate distance could be determined. Taking all these into consideration we need to know the following parameters for a given track segment: initial position, speed and a gear used in the moment when car starts to accelerate at the end of a turn, and parameters of slowing down (braking) to speed that is adequate to the beginning of a turn.

2.1. Selection of track segment

Today, most of racers go through turns as follows. They step on the brake just before a turn and go on braking even in the first segment of a turn. In this stage, deceleration is not so high, as the car must be able to achieve a lateral acceleration that is adequate to the curvature of car trajectory. Afterwards, drivers take the brake off and go through a turn with maximum lateral acceleration. After drivers pass the apex of the turn, they are getting the steering wheel straight and accelerate with partly opened throttle. Finally, they leave a turn at full throttle and go on accelerating along the straight track segment.

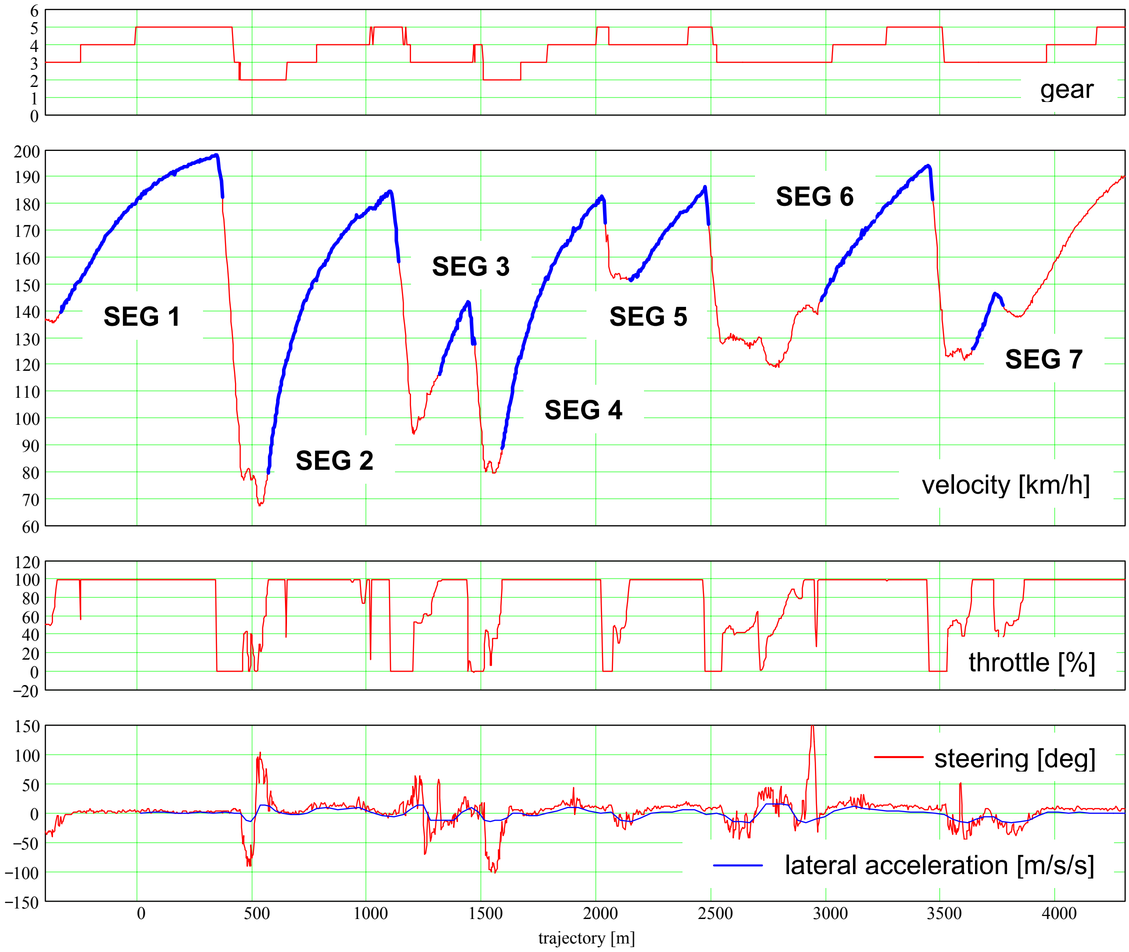


Figure 2: Lap readings from datalogger used at the Autodrome of Most.

Car velocity and gear used at the beginning of selected track segment must be known to perform a simulation calculation. Simulation calculation is based on a maximum acceleration value that can be attained until driver starts to brake before the next turn. That is why it is necessary to locate a point on the track where driver starts to accelerate at full throttle. This point can be identified on the basis of datalogger readings and it is usually found at the end of a turn (see Figure 2). Datalogger readings are used not only for identification of the point, according to which the length of simulated segment is calculated, but also for determination of initial velocity and initial gear.

Selected segment ends after the point where driver starts to brake before the next turn. From the point of view of calculation procedure, it is advisable that the final "braking" part of the segment is as short as possible. Nevertheless, the velocity at the end of segment should be sufficiently low for all calculated gear ratios. In other words, the velocity at the end of segment should be always lower than the velocity, at which the car starts to brake in simulation program. Short distance of braking simulation minimizes the risk of errors that could possibly result from inaccurate value of braking deceleration. End of segment (similarly to the beginning of segment) is deduced from readings of race datalogger. The readings are used also for calculation of distance (length of segment) and velocity (see Figure 2).

2.2. Mathematical model

Longitudinal dynamics of a car can be described by the following equation:

$$\vartheta\, m\, a = F_w k - fmg - \frac{1}{2} c_x \rho\, S_x v^2 - smg \tag{1}$$

After modification for longitudinal acceleration, it can be stated that

$$a = \frac{1}{\vartheta\, m}\left(F_w k - fmg - \frac{1}{2} c_x \rho\, S_x v^2 - smg \right) \tag{2}$$

where ϑ is coefficient of effect of rotational mass, m stands for car weight, F_w is a total driving force on driving axle(s), and k is coefficient of driving force - k is included in the equation in order to describe car movements during gear changes (k=1 for acceleration). f is coefficient of rolling resistance, c_x is coefficient of aerodynamic drag, ρ is air density, S_x is a frontal area of the car, v is velocity, s is road gradient. Second term on the right side of equation (2) is rolling resistance, third term is aerodynamic drag and fourth term is gradient resistance.

Driving force can be calculated from engine torque M.

$$F_w = \frac{1}{r_w} M\, i_t i_f \eta \tag{3}$$

Here r_w stands for wheel dynamic radius, i_t is ratio of the transmission, i_f is ratio of the final drive, and η is a total mechanical efficiency.

Accelerating car is driven by engine that runs at full throttle. Maximum torque that is available in dependence on engine revolutions, i.e. engine external characteristics, is usually stated by engine tuning companies (see Figure 3). For the purposes of our calculation, tabular torque values are interpolated by cubic spline. Engine revolutions for determination of torque will be calculated according to the following relation:

$$n = \frac{1}{c_w} v\, i_t i_f \tag{4}$$

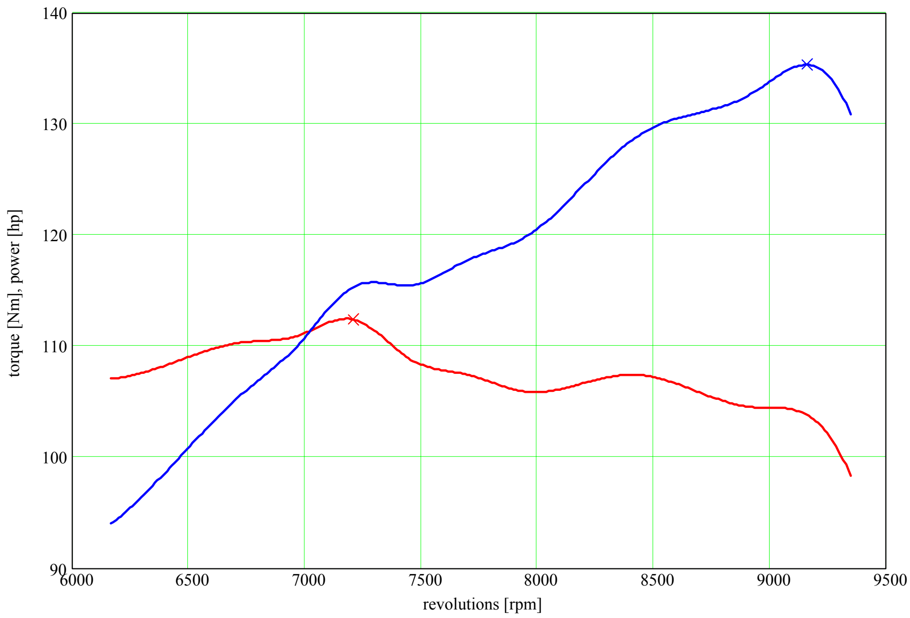
where c_w stands for circumference of a wheel.

Longitudinal acceleration of the car is limited by road adhesion. The following equation applies to rear wheel drive cars:

$$\vartheta\, m\, a = Z_r \mu - Z_f f - \tfrac{1}{2} c_x \rho\, S_x v^2 - smg \tag{5}$$

Z_f, Z_r are vertical forces acting on wheels of front or rear axle; notation of other parameters is similar to equation (1).

max. rpm	rpm at	max. torque	rpm at	max. power
[rpm]	[rpm]	[Nm]	[rpm]	[hp]
9550	7204	112.4	9157	135.33



revolutions	power	torque
[rpm]	hp	Nm
6265	95.5	107.4
6471	100	108.8
6720	105.2	110.3
6977	110	111
7204	115	112.4
7453	115.2	108.8
7700	117.4	107.4
7947	119.6	105.9
8206	124.3	106.7
8432	128.6	107.4
8686	130.7	105.9
8923	132.4	104.5
9157	134.9	103.8
9350	130.6	98.1

Figure 3: Engine external characteristics

Vertical forces acting on wheels of individual axles are determined by static load of wheels on axles, dynamic load induced by inertial mass with centre of inertia in height h above the road surface, weight transfer of axle due to road longitudinal gradient s, and aerodynamic downforce distributed to individual axles:

$$Z_r = m\, p_r g + m\frac{h}{L} a + mg\frac{h}{L} s + \tfrac{1}{2} c_r \rho\, S_x v^2 \tag{6}$$

$$Z_f = m(1 - p_r) g - m\frac{h}{L} a - mg\frac{h}{L} s + \tfrac{1}{2} c_f \rho\, S_x v^2 \tag{7}$$

p_r is a rear axle portion of a total car weight, c_f, c_r are coefficients of aerodynamic downforce acting on individual car axles and L stands for axle base.

The following relation concerning maximum car acceleration is the result of substitution from relations 6 and 7:

$$a_{\max} = \frac{mg\left[\mu\ p_r - f(1-p_r) - s\right] - mg\frac{h}{L}s\ (\mu + f) + \frac{1}{2}\rho\ S_x v^2\left(\mu\ c_r - f\ c_f - c_x\right)}{m\left[\vartheta - \frac{h}{L}(\mu + f)\right]} \tag{8}$$

Simulation calculation is therefore based on solution of a system of ordinary first-order differential equations

$$\dot{\mathbf{x}} = \begin{bmatrix} min(a, a_{\max}) \\ v \end{bmatrix} \tag{9}$$

where $\mathbf{x} = \begin{bmatrix} v & s \end{bmatrix}^T$, a is acceleration calculated according to relation (2), $a_{\max}$ is acceleration determined by equation (8), s is the covered distance and v stands for car velocity.

2.3. Gear changes

Drivers not only accelerate, but also change gears in the selected segment. Simulation program therefore must allow for gear changes too. Simulation program algorithm checks whether the engine reached revolutions sufficient to gear up. Since engine revolutions are not a state quantity, they have to be converted to car velocity:

$$v_s = \frac{n_s c_w}{i_t i_f} \tag{10}$$

n_s stands for rpm that are sufficient for upshift.

Integration algorithm also modifies an integration step in order to reduce the effect of initial step choice on the moment of gear change.

As soon as driver shifts the gear, the program simulates gear change time interval. The following sequence of events should be simulated: driver steps on the clutch pedal, changes the gear and releases the clutch pedal; during the time when the clutch is disengaged, car starts to decelerate; as soon as the clutch is engaged again, car goes on accelerating. The program simplifies this sequence and produces a simulation of reduced or zeroed engine drive torque. Equation of motion is similar to acceleration equation (2), except for the following differences: driving force is reduced; gear is increased by 1; during integration, program checks whether the moment corresponding to the end of gear change was reached:

$$t_{se} = t_s + \Delta t_s \tag{11}$$

t_{se} is time of end of gear change, t_e is time of beginning of gear change and Δt_s is time needed to perform gear change. If necessary, the integration step is modified again.

2.4. Simulation of rev limiter

Rev limiters protect today's race cars from engine overspeeding. After the engine reaches its maximum permissible rpm, speed limiter will not let it accelerate any more, and maintains rpm at maximum level. Similarly to gear change calculation, the program checks whether the car reached velocity v_{nMax} that corresponds to maximum engine revolutions, i.e. “speed limiter rpm” n_{Max} :

$$v_{nMax} = \frac{n_{Max} c_w}{i_t i_f} \tag{12}$$

Afterwards, system of equations of motion corresponding to constant engine revolutions is being solved:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 \\ v \end{bmatrix} \tag{13}$$

2.5. Brake action

At the end of straight segment before the next turn, driver must slow down (brake) to speed that allows him to negotiate the turn. Driver should start to brake as late as possible and slow down in the shortest possible distance in order to achieve the best lap time. In this sense, the simulation algorithm should calculate (for every velocity and position of the car) the theoretically shortest distance needed for slowing down to required speed (input value of the calculation), and to check whether requested segment length (input value) is exceeded or not. Since the input values were selected in a way that allows modelling of only short braking segment, braking deceleration can be simply considered as constant. Distance s_b in which the car slows down from velocity v_b to velocity v_e is calculated according to

$$s_b = -\frac{v_b^2 - v_e^2}{2a_b} \tag{14}$$

where a_b stands for longitudinal acceleration during brake action. a_b is negative. Equations of motion during brake action have the following form:

$$\dot{\mathbf{x}} = \begin{bmatrix} a_b \\ v \end{bmatrix} \tag{15}$$

Time needed to slow down is determined by the following relation:

$$t_b = -\frac{v_b - v_e}{a_b} \tag{16}$$

If the simulation calculation is not used to calculate velocity behaviour in braking segment of the track, a total time needed to cover selected segment can be obtained by adding the time calculated via equation (16) to the time of beginning of brake action. If so, it is not necessary to solve equations of motion (15).

2.6. Application of the method

A procedure for calculation of time needed to cover one segment of straight track has been described above. It can happen that optimisation of gear ratios leads to shorter time in one segment and longer time in another. In such cases, it is the lap time that is relevant. So it is necessary to count up all segment times and evaluate the overall time reduction (or extension).

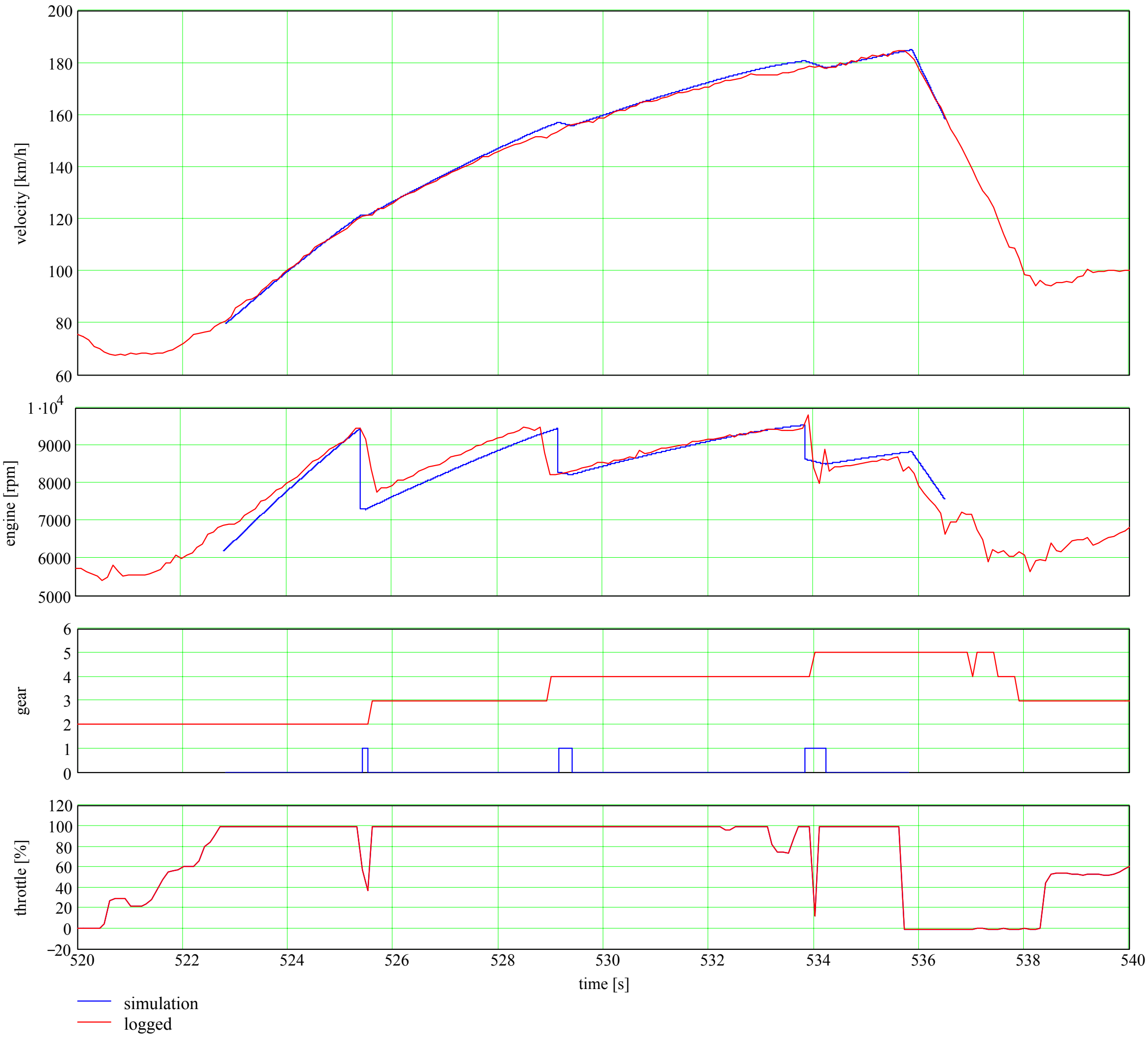


Figure 4: Comparison of datalogger readings with simulation of segment 2

Segment data necessary for calculation are incorporated into a matrix where each line corresponds to one segment. Each line includes the following data: velocity, distance and gear at the beginning of segment; final segment distance and velocity; segment gradient.

Calculation accuracy depends on accuracy of various input data: engine torque curve as function of rpm, aerodynamic drag coefficient, driveline efficiency, rolling resistance, road gradient etc. In fact, many input data are not measured, but estimated (e.g. road gradient, aerodynamic drag coefficient for different combinations of front and rear wing angle, relative position of car body and road surface etc.). It is very difficult to align the calculation procedure with real situation on the track. Still before calculation, it is necessary to tailor parameters of the model to data logged during car operation. To do this, the input data must be structured in a way that permits tailoring of simulation to racer's driving behaviour in the monitored segment. For these purpose engine revolutions logged at the moment when driver engages individual gears, are recorded to a matrix. Lines correspond to particular track segments, columns show rpm at which the driver engaged appropriate gear. Similar matrix is developed for appropriate times of gear change duration.

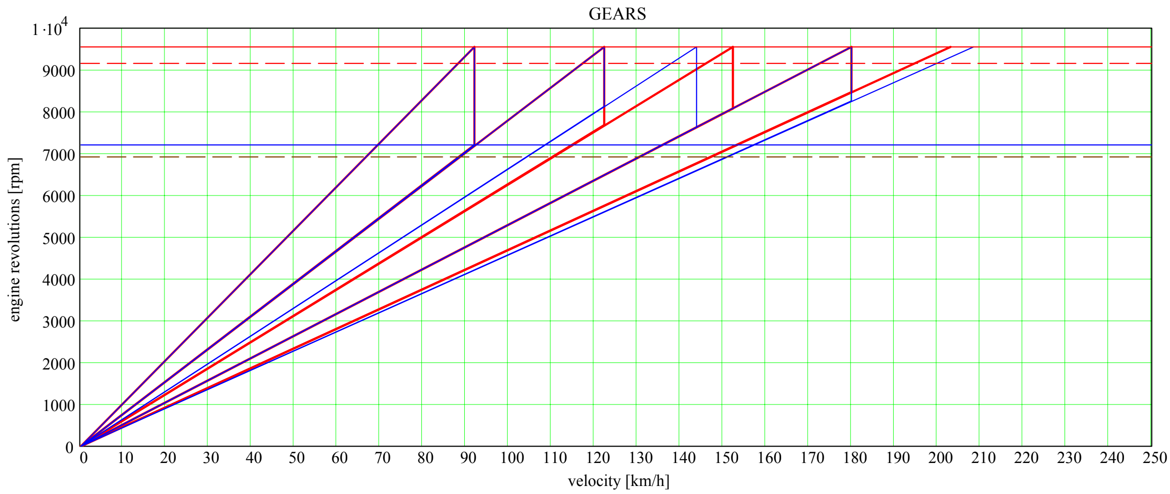


Figure 5: Sawtooth diagram - version 1 (red); version 2 (blue)

Optimisation calculation is performed manually in this stage. The main calculation criterion is a total time needed to cover all segments. Gear ratios for individual gears and final gear ratio in axle drive are being optimised. As only limited number of gear sets is available, it is faster to calculate all their combinations (and select combination with the best time) than to use numerical optimisation procedure right away.

The method is demonstrated by the results of two versions of gear-ratio stepping. Sawtooth diagram of both versions is showed in Figure 5. Results of simulation calculations for 7 segments of the Autodrome in Most are demonstrated in Table 1 and Table 2.

Table 1: Calculation results of gear-ratio stepping, version 1

		Beginning of segment			End of segment		
Segment number	Time [s]	Gear [1]	Velocity [km/h]	Engine [rpm]	Gear [1]	Velocity [km/h]	Engine [rpm]
1	14.319	3	139.6	8716	5	198.1	9550
2	13.569	2	79.6	6186	5	188.6	9550
3	4.196	3	116.2	7255	3	144.7	9036
4	10.700	2	88.6	6891	5	187.1	9550
5	7.146	4	151.7	8011	5	187.2	9550
6	10.077	3	143.7	8972	5	193.9	9550
7	3.381	3	125.8	7854	4	155.0	9550
All segments	Total time 63.388		Min. 79.6	Min. 6186		Max. 198.1	Max. 9550

Table 2: Calculation results of gear-ratio stepping, version 2

		Beginning of segment			End of segment		
Segment number	Time [s]	Gear [1]	Velocity [km/h]	Engine [rpm]	Gear [1]	Velocity [km/h]	Engine [rpm]
1	14.404	3	139.6	9228	5	197.0	9550
2	13.593	2	79.6	6186	5	188.0	9550
3	4.170	3	116.2	7681	4	144.5	9550
4	10.717	2	88.6	6891	5	186.6	9550
5	7.148	4	151.7	8011	5	186.9	9550
6	10.134	3	143.7	9499	5	193.0	9550
7	3.385	3	125.8	8316	4	154.2	9550
All segments	Total time 63.551		Min. 79.6	Min. 6186		Max. 197.0	Max. 9550

3. Conclusion

Calculations based on procedure described in this article prove that appropriate arrangement of gear ratios can reduce lap times about tenth of second. And this can have a substantial impact on standing in qualification or even race.

Simulation of only straight segments of racing track significantly reduces a number of input data (parameters of both car and track) needed for calculation. At the same time, the calculation is relatively fast, as it concentrates only on segments that are most affected by selection of gear ratios.

Our experience indicates that the overall calculation could be much more accurate, if we had precise track gradient data. Just one gradient value for the whole segment (straight section) is not enough. Also, it seems that the program based on the method described above could be used more efficiently, if we had a procedure for automatic adjusting of car parameters to behaviour recorded during car operation. For that reason, we are going to aim our efforts particularly to these problems.

4. Acknowledgments

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5. Literature

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