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Simulation of Fracture Process Using Spring Networks

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Summary: *The paper deals with a fracture simulation of a concrete. A specimen volume is discretized using two different types of spring networks - a truss lattice and a rigid-body-spring network. The first model is valuable for its simplicity, nodes are interconnected by ideally brittle springs bearing only an axial force. The paper shows that such model is not suitable for more difficult type of load (mixed-mode experiments). The second type, the rigid-body-spring network, is able to simulate fracture process even for mixed-mode experiments and does not loose a physical explanation for any part of modeling procedure. Unfortunately, both models exhibit a mesh size dependency, which is described in the paper via an extensive parameter study.*

1. Introduction

A modeling of a fracture in disordered materials (like concrete) is under intensive research during last decades. Lattice simulations appear as a promising approach focused primarily to obtain a correct shape of a crack. Since the material structure is included in a model directly, one therefore can use very simple constitutive laws. Particulary these spring network models used to have originally brittle constitutive law for every spring. With such constitutive law one can simulate the crack pattern but to fit an experimental response, many researchers adopted softening (Berton and Bolander (2006), Ince et al. (2003)). Other progress has been done from mechanical point of view. The truss lattices were substituted by beam lattices or rigid-bodies . Moreover, many two dimensional lattice models were recently adapted into three dimensions (Yip et al. (2005), Lilliu and van Mier (2003)).

The paper is concerned with modeling of plain concrete at meso-level that provides a realistic simulation of a fracture process in a laboratory specimen scale. In particular, two types of a model are studied: a truss lattice model (beams carry only normal forces) and an assembly of rigid cells interconnected by normal and shear springs. This concept was originally proposed by Kawai (1978). Both models have an irregular (random) geometry and spring properties are assigned by overlapping the model with a structure

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of aggregates generated randomly according to given grain size distribution. The original aim of such models is not to match experimental data or to find material parameters. The most important goal is the ability to simulate a real fracture process with simple brittle constitutive laws in a step-by-step linear calculation. A nonlinearity of a fracture in concrete is included directly inside the model via the overlapped aggregate structure.

2. Modeling framework

Both models are built in a similar way - first nodes of lattice are generated and interconnected by springs, then the aggregate structure is used to determine material phases and parameters of these brittle springs. Finally a function evaluate spring strains from nodes displacements has to be determined.

The nodes are generated pseudo randomly after Moukarzel and Herrmann (1992) to decrease mesh shape dependency (van Mier (1997)). A domain is divided into rectangular cells of size s and subsequently one node is randomly chosen chosen for each cell. A randomness of a node position can be controlled by the parametr $t \in \langle 0, s\rangle$ (fig. 1). When $t = 0$ regular node positions are obtained. A node connectivity is determined by Delaunay triangulation. This triangulation provides triangles with the largest inner angles. Many fast algorithms have been invented to generate this triangulation (for instance Sloan (1993)). In case of the truss lattice, each edge of the triangulation represents one normal spring. When the concept of rigid bodies is used shear and normal springs are placed into the model instead. To ensure elastically uniform lattice (Schlangen and Garboczi (1996), Bolander and Saito (1998)), we use Voronoi tessellation to find areas of springs.

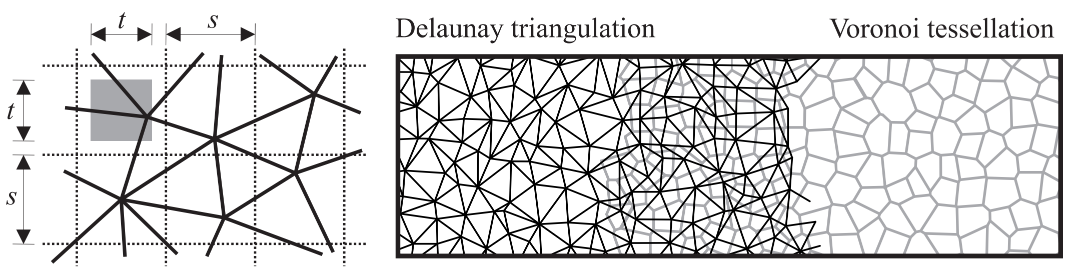

Fig. 1: Left: a generating of the network nodes (according to van Mier (1997)). Right: an example of Delaunay triangulation and Voronoi tessellation of the domain.

A material inhomogeneity is applied by an overlapping aggregate structure. This structure could be obtain by scanning material surface or generated in a computer. An algorithm used in this paper to generate aggregate structure has two main steps: the former step consists of calculating numbers of grains with certain diameters. The latter involves placing grains into the volume. Amounts of grains are given by Fuller curve $F_d = (d/d_{max})^{0.5}$ (see fig. 2). One has to determine maximal diameter d_{max}, usually according to real batch grain contents. The minimal diameter d_1 is set out together with a mesh size (the size of one cell s in the node placing algorithm) because after van Mier (1997) the size of the mesh should be at least three times smaller then the minimal aggregate diameter. Then the random sequential addition (Widom (1966)) is used to find positions of grains in a specimen volume.

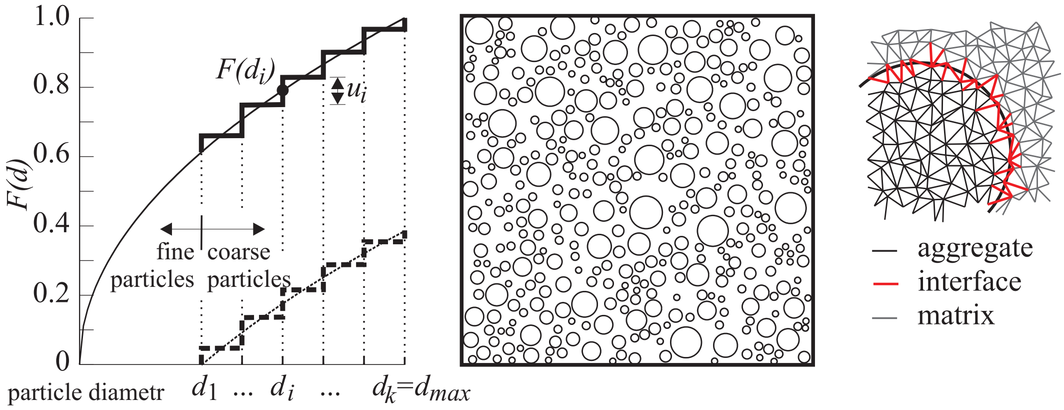

Fig. 2: Left: Fuller curve provides an amount of grains for certain diameters (after Cusatis et al. (2006)); middle: aggregates are placed inside a specimen via random sequential addition; right: three material phases are determined according to node coordinates.

When geometrical parameters of the spring network and the aggregate structure are determined material parameters of the springs can be set out. Three material phases are distinguish: aggregate, matrix and interface. The spring having both nodes in matrix are classified as the matrix phase, in the same way the aggregate phase springs are found. Springs with one node in the matrix and second one inside some aggregate come under the third interface phase.

Material parameters of phases were determined with a help of literature. Lilliu and van Mier (2003) suggested elastic moduli E and tensile strength f_t of phases and ratio between these suggested numbers has been verified by measuring acoustic emissions during fracture process (Bolander et al. (1998)). The material parameters used in this paper are: $E_a = 70$ GPa; $E_m = E_i = 25$ MPa; $f_{t,a} = 24$ Mpa; $f_{t,m} = 12$ Mpa; $f_{t,i} = 4$ Mpa. Also shear modulus E_s and shear strength f_s have to be determined in case of the rigid-body-spring network (here $E_s = E/4$ and $f_s = 3/2 f_t$ for all phases).

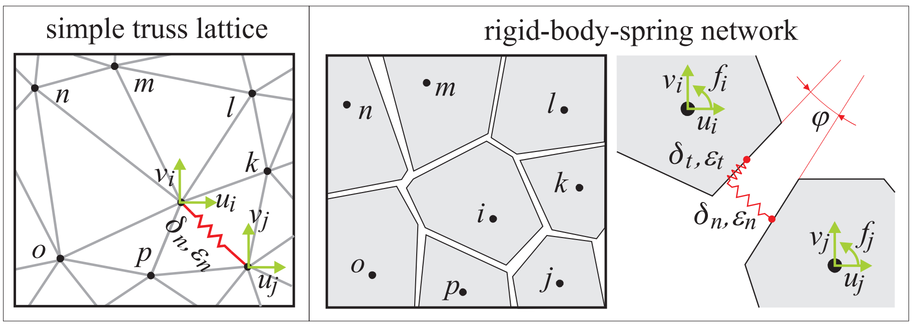

Fig. 3: Nodes of lattice interconnected by springs. Left: the truss lattice, two degrees of freedom per each node; right: the rigid-body-spring network, three degree of freedom per each node.

The following section explains the main difference between considered lattice types. The truss element bears only normal strain ε_n that can be simply evaluated using an actual change of spring length $\varepsilon_{n,k} = \delta_{n,k}/l_k$. The length l_k is an original length of spring k and the actual length change $\delta_{n,k}$ is obtained from nodal displacements u and v (fig. 3).

However, the concept of rigid-body-spring network includes nodal rotations. The displacements $\delta_{n,k}$ and $\delta_{n,k}$ of element k are evaluated according equations 1, 2, which are derived from a motion of the rigid bodies. The position of both normal and shear springs is assumed in the middle of Voronoi tessellation edge (fig. 4).

$$\delta_{n,k} = (u_j - u_i) \cdot \cos\alpha_k + (v_j - v_i) \cdot \sin\alpha_k + (f_i - f_j) \cdot p_k \quad (1)$$
$$\delta_{s,k} = -(u_j - u_i) \cdot \sin\alpha_k + (v_j - v_i) \cdot \cos\alpha_k - (f_i - f_j) \cdot l_k/2 \quad (2)$$

Strains are then evaluated in ordinary way $\varepsilon_{n,k} = \delta_{n,k}/l_k$, $\varepsilon_{s,k} = \delta_{s,k}/l_k$.

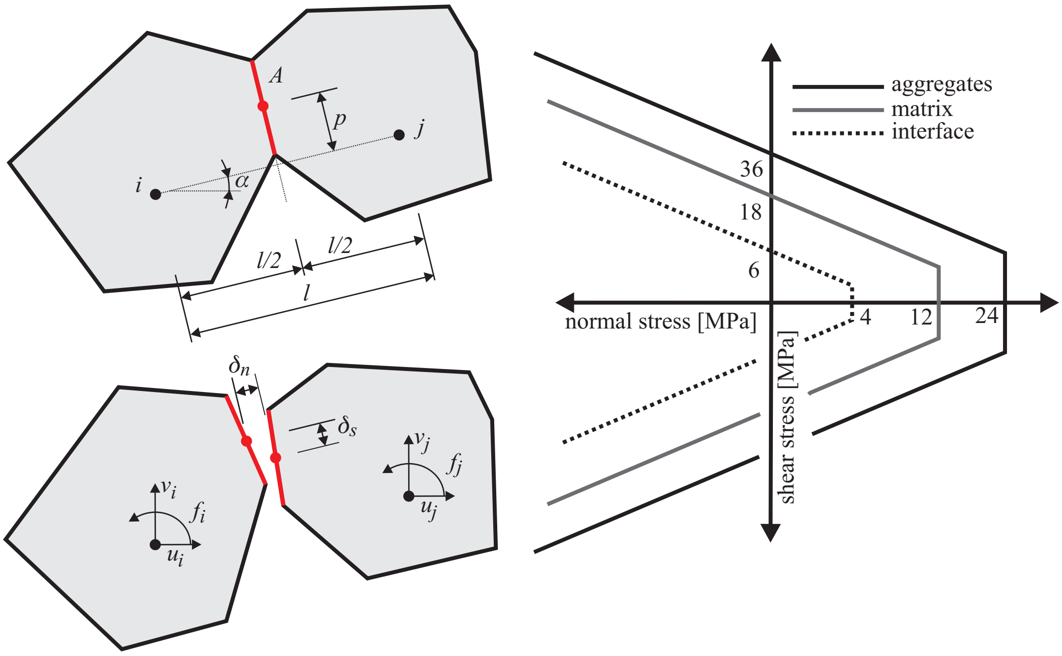

Fig. 4: Left: the motion of rigid bodies, variables description; right: the elemental breaking envelope for different material phases.

A solution of fracture process proceeds in events. In each event one element breaks. In order to find this element, one needs breaking condition (envelope) to determine utilization of each connection. In case of the truss lattice, the element breaks when the normal stress is higher then the tensile strength ($\sigma_n > f_t$). The breaking envelope for the rigid-body-spring network is plotted in fig. 4 right. The normal and shear stresses are confined.

The global stiffness matrix is built on the beginning of solution process. Then task proceeds in linear steps (events). One step of the solution has five points:

- The load is applied by a vector of prescribed initial displacements and initial nodal displacements are calculated. The solving of this system of linear equations is a crucial point because of time consumed on this row of a code implementation. So advanced iteration techniques are employed, in this case the preconditioned conjugate gradients method.

- Subsequently the initial strains and stresses are evaluated. Then utilization ξ of each element is calculated according to breaking conditions.
- The element $e_{\xi,max}$ with maximal utilization ξ_{max} is found.
- The whole solution (the initial nodal displacements, the initial nodal forces) is divided by utilization ξ_{max} to ensure that the stress state of edge $e_{\xi,max}$ will lie exactly on the envelope curve.
- The element $e_{\xi,max}$ is remove from stiffness matrix (crack propagation).

Except of a sequence of broken elements one can record also a response of the model. In this paper an overall displacement (displacement between supports) is used to present response of virtual specimens. Obtained load-displacement curve consists of many linear steps which has to be smooth (fig. 5).

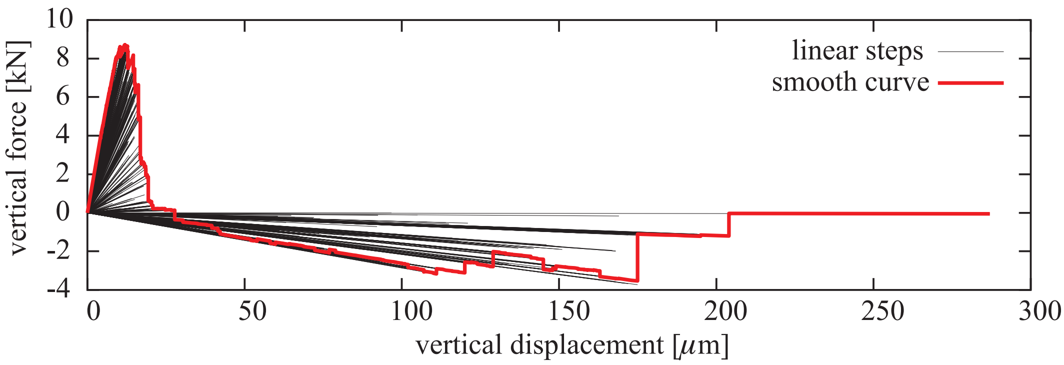

Fig. 5: The response consists of many linear steps. Only steps with increasing prescribed displacement are accepted.

3. Applicability for uniaxial tensile test and mixed-mode test

Almost no articles dealing with the truss lattice models are published today. A general reason for that is that networks built from beams bearing bending moment give more realistic crack pattern (Schlangen and Garboczi (1997)). Using beams bearing bending moment at the concrete meso-level structure has no physical explanation. The rigid bodies are able to transfer bending moment with physical background. The truss lattice is a logical choice to begin with lattice simulations and that is the reason to mention it here.

Figure 6 shows fit of the uniaxial tensile test with pure truss lattice. Unfortunately, experimental data are damaged probably because of incorrect measuring device setting. The ability of the truss lattice to provide right response curve shape is obvious even when recorded displacements are unrealistic high. This quasi brittle response was obtained only by cracking ideally brittle truss elements.

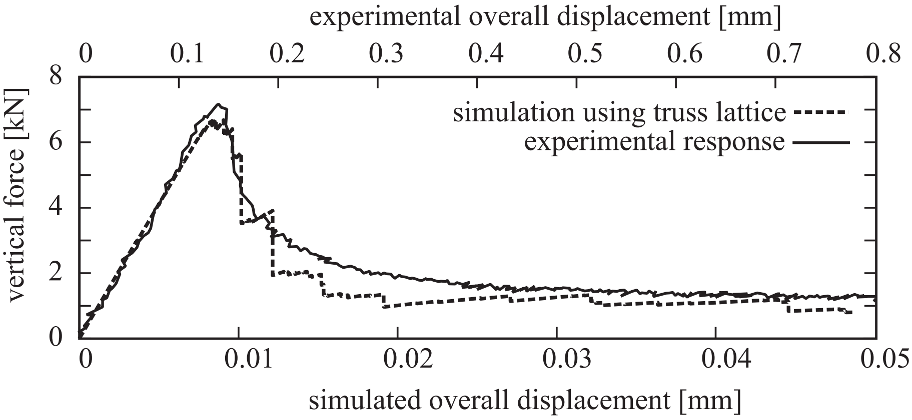

Fig. 6: A record of the uniaxial tensile test and a corresponding simulation using the truss lattice model. A discrepancy between displacements measured during experiment and simulated displacements is probably caused by an incorrect setting of measuring device.

Limits of the truss network are shown on mixed-mode experiments. The truss lattice is not capable to describe the real crack propagation (two angled cracks from both notches). The rigid-body-spring network works well in that case - see fig. 7.

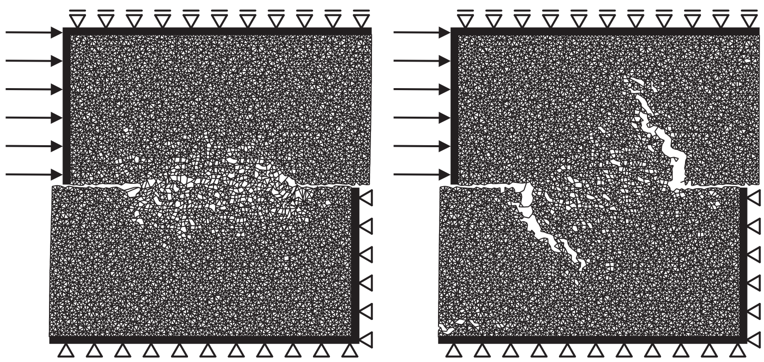

Fig. 7: An ability to simulate the crack pattern for two lattice model types. Left: the truss lattice; right: the rigid-body-spring network.

Mixed-mode tests from literature (Nooru-Mohamed (1992), load path 6a) were simulated using rigid bodies. The concrete batch included only aggregates with a diameter less then 2 mm. But aggregates with diameters $d \in \langle 4\text{mm}, 2\text{mm}\rangle$ were generated because of computational difficulties. Otherwise the network would be too fine and a computational time extremely long. Three specimen sizes were considered, the large specimen $200 \times 200 \times 50$ mm with a notch 25 mm on both sides, the middle specimen $100 \times 100 \times 50$ mm with a notch 12.5 mm on both sides and the small one $50 \times 50 \times 50$ mm with a notch 6.25 mm on both sides. These specimens were loaded by prescribed normal and shear displacement in the rate of 1:1 simultaneously. Different mesh sizes were used. Because of computational time only coarse network was applied for large specimen. Responses and crack patterns are plotted in figure 8. Lattice responses are too brittle in comparison with real responses but crack shapes and an occurrence of vertical compressive force are in good agreement.

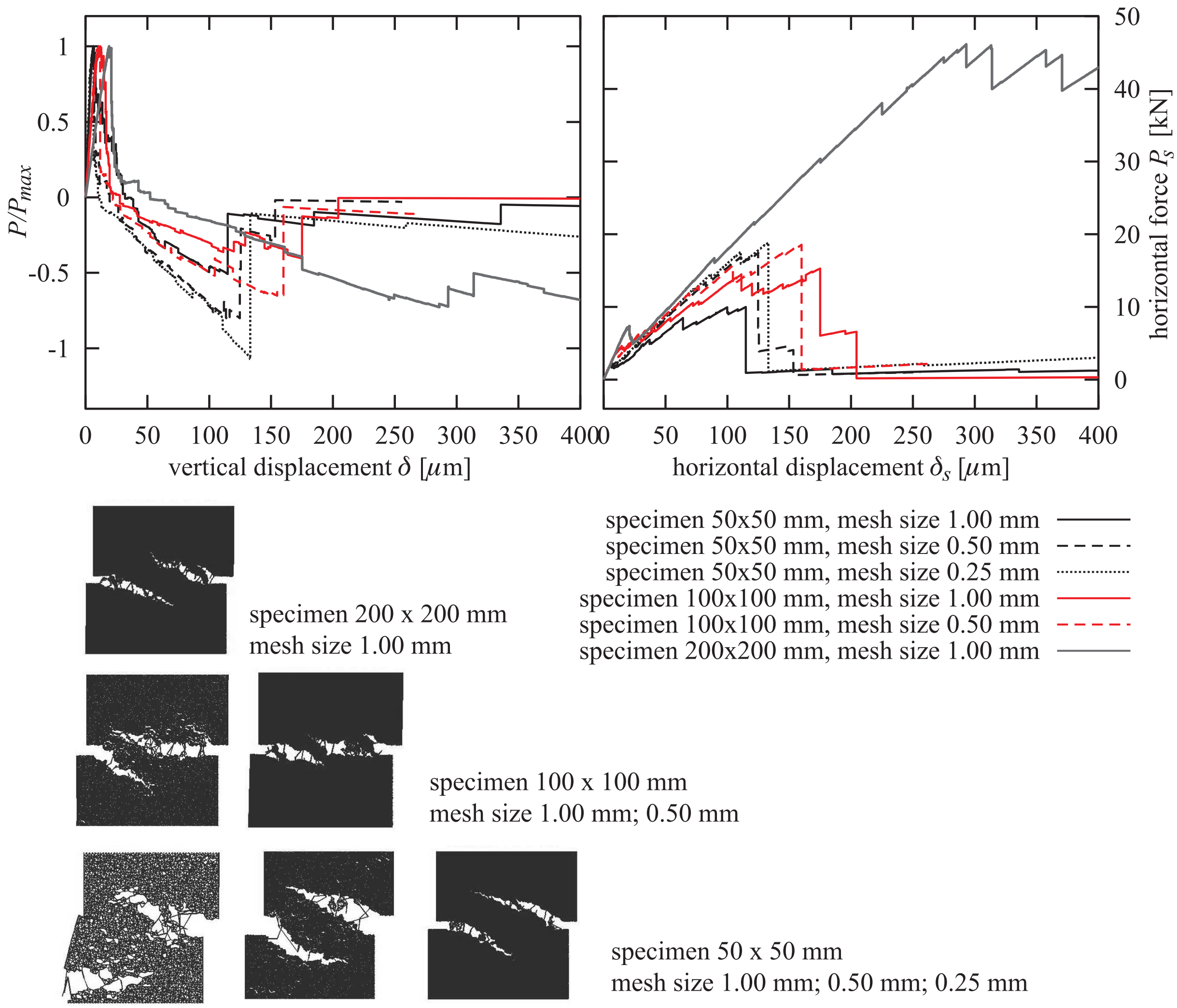

Fig. 8: Responses and crack patterns of virtual specimens subjected to the mixed-mode loading according to Nooru-Mohamed (1992).

4. Network size dependency

Perhaps the most obvious disadvantage of described lattice approach is a network size dependency. Because the solution proceed in discrete steps a change of one broken element can cause entirely different following sequence of broken elements. The other source of this dependency is a representation of stress distribution in notch areas. A coarse mesh leads to stress averaging but in case of fine mesh stress is concentrated in small area close to the notch. In order to describe the mesh dependency several simulations of the uniaxial tensile test have been done. A specimen size were constant $50 \times 50 \times 50$ mm with noth 6.25 mm on both sides, aggregates diameters were in range from 6 mm to 2 mm. The aggregate structure were kept the same in all cases. Five network sizes were used: 1.00 mm, 0.50 mm, 0.33 mm, 0.25 mm and 0.20 mm. The coarsest mesh 1 mm strongly breaks size condition (mesh size should be at least one third of the smallest grain diameter). Responses and obtained crack patterns are presented in figures 9 and 10.

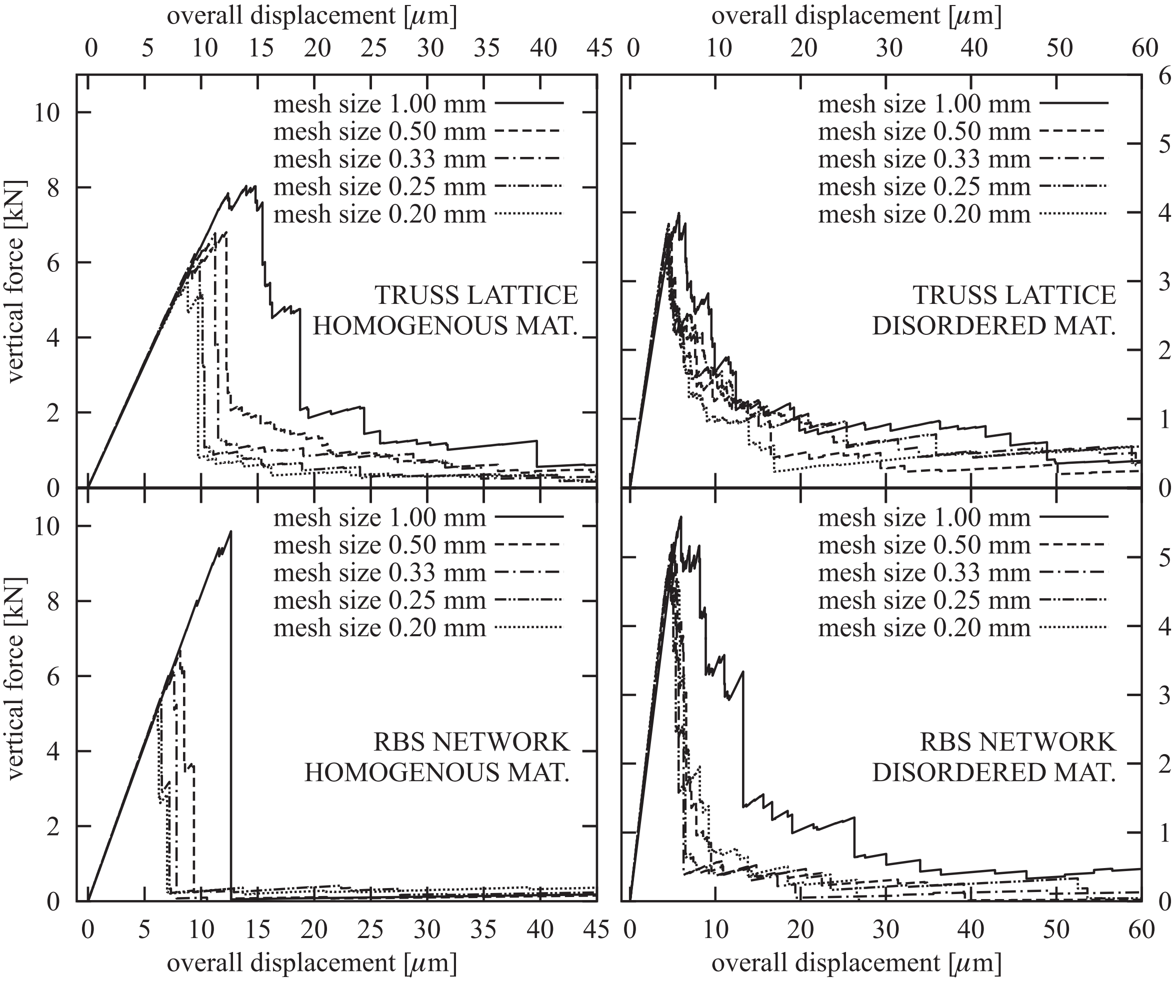

Fig. 9: Responses measured during uniaxial tensile test simulations for various network sizes. The disordered material includes aggregate structure which is identical for all simulations.

Several conclusions can be done with help of this study. Both types of lattice models exhibit strong mesh size dependency in case of homogenous material which is probably caused by the stress representation around notches. The reason for this conclusion is that disordered material decrease this dependency. Perhaps because the aggregate structure has the main influence on crack propagation and notch effect diminishes. Unfortunately other complication appears in case of disordered material. It is obvious that a crack shape for identical aggregate structure depends on mesh size. That is again caused by different representation of stresses, now around grains. Study also displays another problem. The model works on concept of small deformations which can caused unrealistic fracture propagation (fig. 10, mesh size 0.20 mm, rigid-body-spring network, homogenous material). Elements with "bad" orientation can bear more load than others. Finally an interesting effect is visible in case of homogenous material for truss lattice. Unrealistic crack shape for network size 1 mm becomes better for finer meshes. It looks also that response converges to some certain solution. This points to examine abilities of extremely fine truss lattices. But such examination runs against an extreme computational effort.

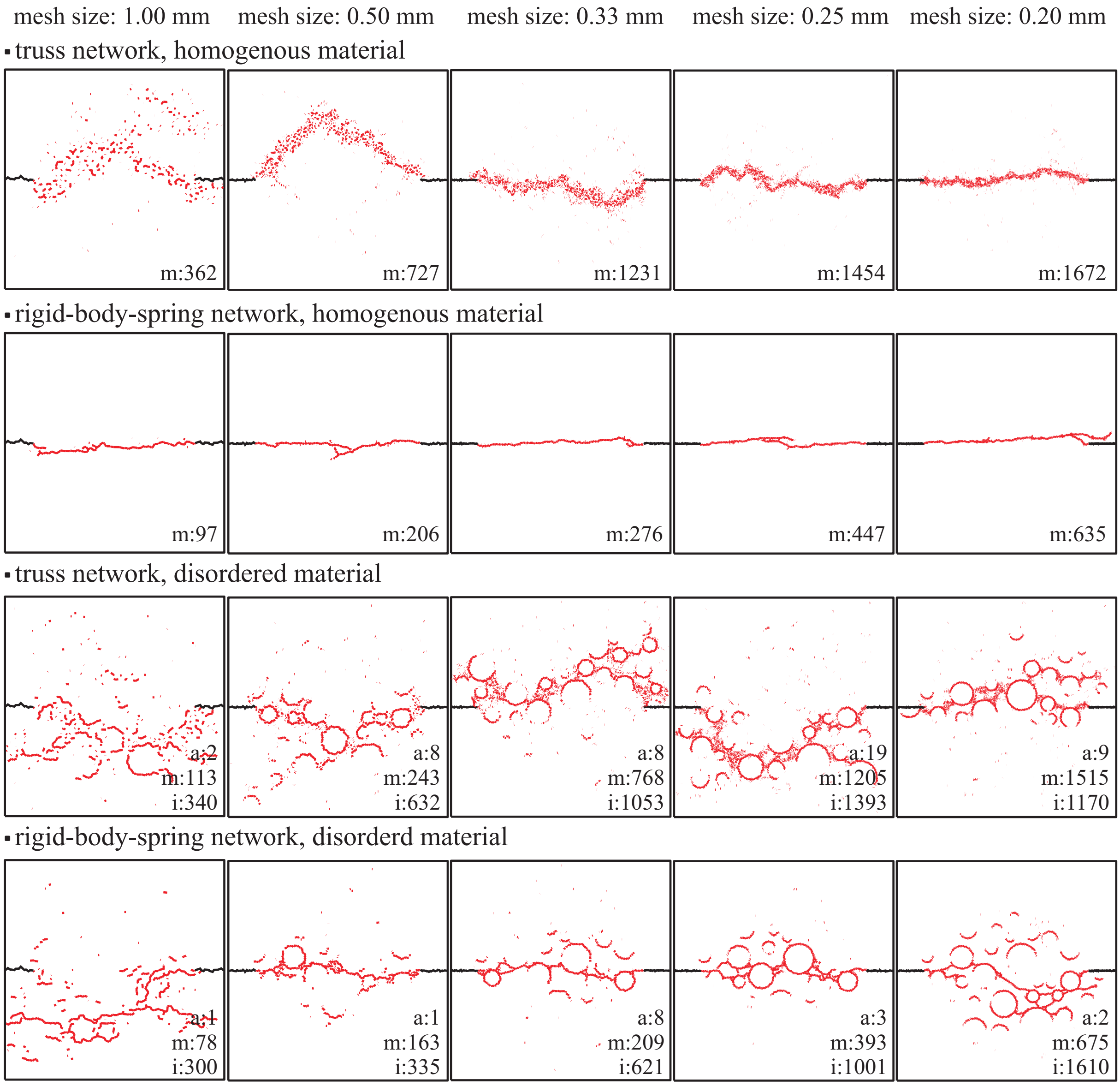

Fig. 10: Crack patterns of virtual specimens subjected to uniaxial tension. Numbers of cracked elements from each phase are plotted.

5. Aggregate structure influence

An influence of aggregate structure has been studied as well. A set of grains was generated and five different positions of grains from that set were found. The results are plotted in figures 11 and 12. Response differences are within an expected scatter.

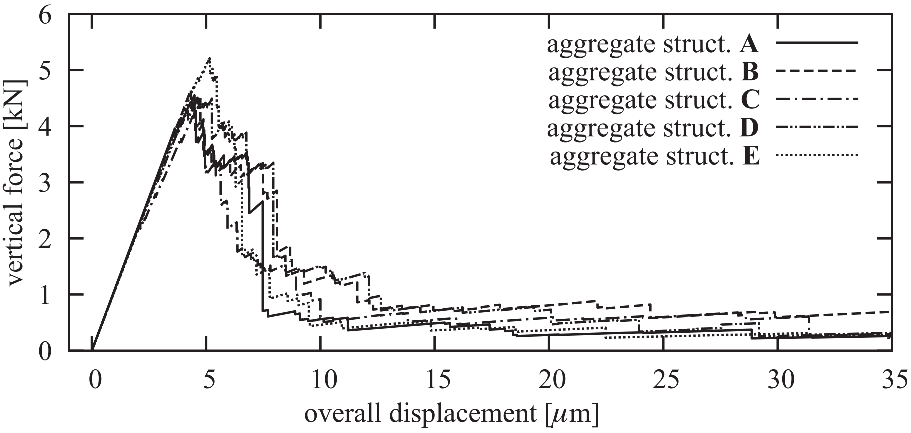

Fig. 11: Responses measured during uniaxial tensile test simulations for various aggregates structures. Amounts of grains and their diameters are identical in all cases.

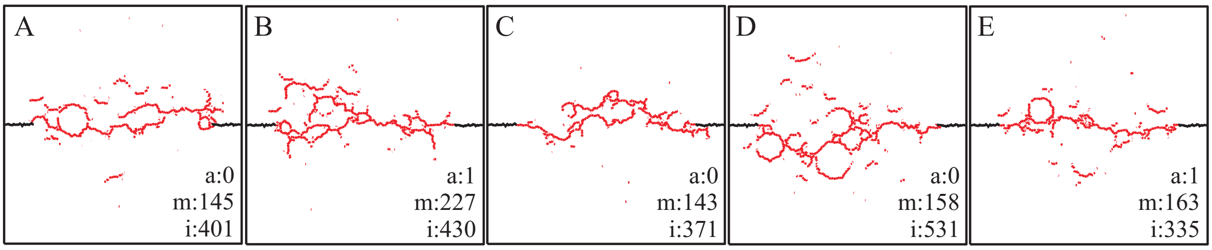

Fig. 12: Crack patterns of virtual specimens subjected to uniaxial tension with variable grain positions. Numbers of cracked elements from each phase are plotted.

6. Conclusions

The paper shows the ability of lattice models to simulate a fracture process. A particular description of the truss lattice model and the rigid-body-spring network is presented. A capability to simulate the uniaxial tensile test and the mixed-mode test is verified. The simple truss model is able to represent the tensile tests; however one cannot obtain a correct crack pattern for mixed-mode tests. On contrary, the rigid body network model works well even in the mixed-mode situation. Also the mesh size dependency and the influence of aggregate positions were examined and described.

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