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FACTORS INFLUENCING THE ACCURACY OF THE MEASURED STRESS-STRAIN CURVES OF ELASTOMERS

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Summary: *This article deals with three different factors which can influence the accuracy of the stress-strain data of elastomers measured with a biaxial testing rig. In terms of computational simulations of experiments the influence of: spacing between clamps, dimension of the specimen and distance of reference length was studied. Simulations showed that some factors have substantial influence on measuring data. The obtained information was used for improvement of the accuracy in real experiments realized on the mechanical testing machine Camea.*

1. Introduction

Constitutive behaviour of hyperelastic isotropic materials (e.g. elastomers, biological soft tissue) is derived from the strain energy density function. Determination of material parameters occurring in the isochoric part of the strain energy density function often requires several different kinds of material tests: uniaxial, equibiaxial and planar tension tests. All of these tests can be carried out at the mechanical testing machine Camea (fig. 1), which was primarily proposed for biaxial tests of elastomers or arteries (those immersed in the physiological solution). During the tests, forces are measured through two independent force sensors (x and y direction) and a camera (placed above the specimen) makes numerous photos of the deformed specimen with points on its surface. The stretch ratio of the specimen is then evaluated from the photographs as a ratio of the distance between points in the current and reference photographs.

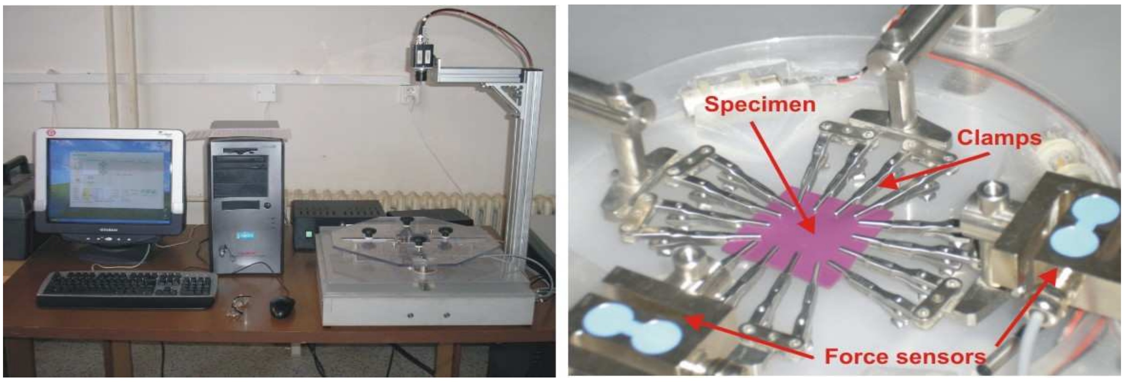

Figure 1 Mechanical testing machine Camea

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Several factors influencing the accuracy of the measured data are evaluated in this paper, particularly spacing between clamps, dimensions of the specimen and reference distance between points on the specimen top surface. The main objective is to ascertain whether some of these factors have substantial influence on the measured data, and to determine optimal testing configuration for each kind of test. For this purpose, computational simulations of experiments were carried out, the influence of each factor was evaluated and optimal testing configuration was proposed. Next, experiments with real specimens were carried out for the optimal testing configuration and for another one.

2. Computational simulations of experiments

In all simulations the incompressible Arruda-Boyce (1993) constitutive model was used for the tested specimen. It is a statistical model, in which the material parameters are physically linked to the elongation of molecular chains involved in the three-dimensional network structure of the rubber. The strain energy density function W is derived by means of Taylor's expansion of inverse Langevin function; the first five terms have been used in our simulations. Material parameters are the initial shear modulus $\mu = 1$MPa and the limiting network stretch $\lambda_L = 5$. For more information about the Arruda-Boyce model see e.g. Holzapfel (2000).

From every computational simulation Cauchy stress and stretch ratio were calculated in the same manner as in the experiment on the testing machine Camea. It means that stretch ratio in the direction of loading was determined according to the following equation:

$$\lambda = \frac{L0 + u}{L0} \ , \tag{1}$$

where $L0$ is the reference distance between points on the specimen surface and u is the difference in displacements of these points in the direction of loading. The corresponding Cauchy stress in the direction of loading was calculated according to the following equation:

$$\sigma^C = \frac{F}{L.T} . \lambda \ , \tag{2}$$

where F is the force measured by the force sensor and L is the width and T the thickness of the specimen in its reference (undeformed) configuration.

For the stretch ratio determined by equation (1), theoretical Cauchy stress σ^C_{theo} in the loading direction was then calculated by using the following equation:

$$\boldsymbol{\sigma}^C_{theo} = -p\mathbf{I} + \frac{\partial W}{\partial \mathbf{F}} \mathbf{F}^{\mathbf{T}} \ . \tag{3}$$

In the equation (3) **F** is a deformation gradient, W is the Arruda-Boyce strain energy density function (which depends on the deformation gradient) and p is a Lagrange multiplier introduced due to the incompressibility. When we have written equation (3) in the principal direction (separately for each kind of test), we got an expression for the theoretical Cauchy stress in the direction of loading as a function of the stretch ratio λ and the material parameters.

In order to evaluate the influence of the factors mentioned in introduction, the deviation d between Cauchy stress (2) and the theoretical Cauchy stress (3) was calculated as follows:

$$d = \frac{\sigma^C - \sigma^C_{theo}}{\sigma^C_{theo}}.100\,. \qquad (4)$$

It should be noted that if no factors have any influence to measured data, Cauchy stress (2) and theoretical Cauchy stress (3) must be identical and the deviation (4) equals zero.

2.1 Influence of spacing between clamps

First, the influence of spacing between clamps was studied. Three different values of spacing *b* between clamps were considered, i.e. 5 mm, 8mm and 12.5 mm for each type of test (uniaxial, equibiaxial and planar tension tests). The specimen used in simulations had dimensions 50x50 mm, thickness T = 1 mm, and the reference distance of the points was L0 = 8 mm. In case of the uniaxial tension test, the two opposite sides had four clamps (figure 2a)); in other cases four clamps were on each side of the specimen (see figure 2b)). Corners of the area L0xL0 represent points used for calculation of stretch ratio in equation (2).

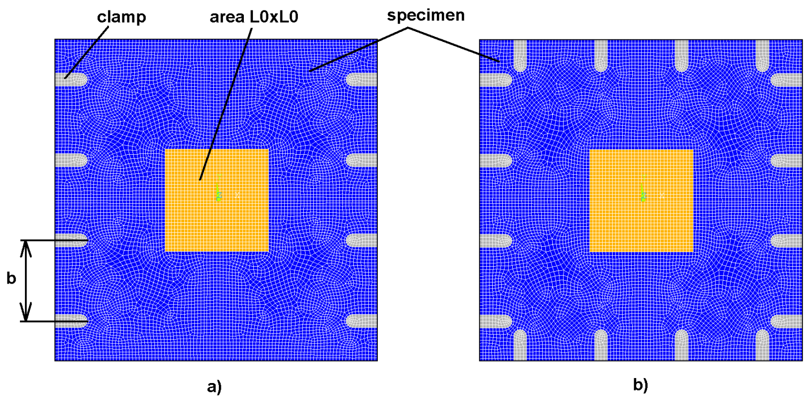

Figure 2 Computational models of the test configuration used in the simulations

In terms of the computational simulations, the stretch ratio (2) and the Cauchy stress (3) were calculated for each type of the test and for the three different spacings between clamps. In the following three figures the dependence between deviation of stress (4) and stretch ratio (2) is shown for the uniaxial, equibiaxial and planar tension tests. We can see that the largest deviations occur in the cases with the smallest spacing b = 5mm and minimum deviations are in the cases with the largest spacing b = 12.5mm. When the specimen with spacing b = 12.5mm was used, the stretch ratio and the stress almost all over the length of the specimen were constant, while maximum stretch ratio and stress are situated in the middle of the specimen in the case with the smallest spacing b = 5mm. The difference between the stresses (and stretch ratios) calculated in the middle part and near the ends is pronounced. Then the Cauchy stress calculated by equation (2) is an average stress through the width of the specimen but it includes the maximal stretch ratio from the middle part of the specimen. Hence, the deviation between stresses must increase with the decreasing spacing between clamps, because the stress state in the specimen becomes non-homogeneous; therefore the best spacing between clamps is the largest one (b = 12.5mm).

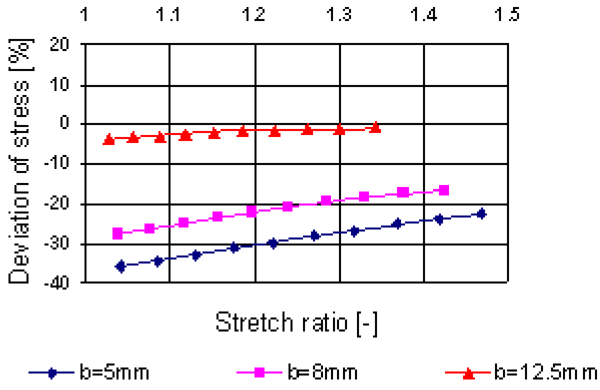

Figure 3 Influence of spacing between clamps in case of the uniaxial tension test

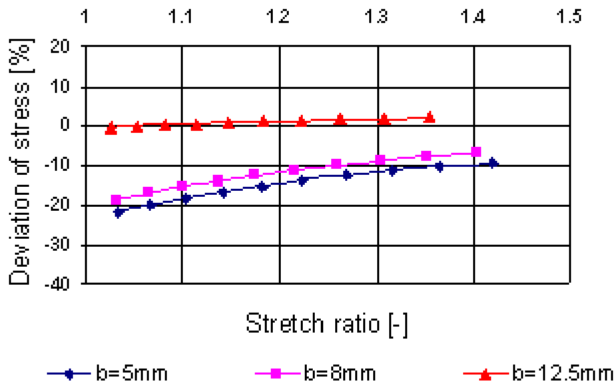

Figure 4 Influence of spacing between clamps in case of the planar tension test

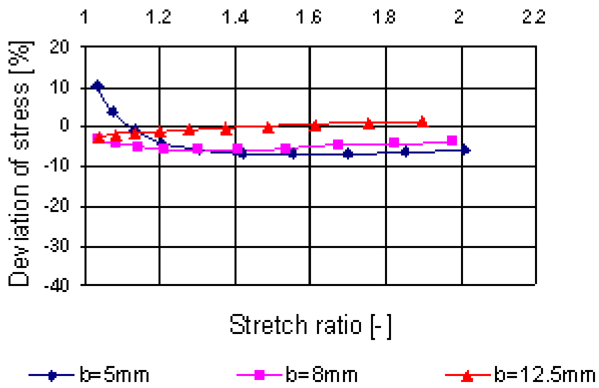

Figure 5 Influence of spacing between clamps in case of the equibiaxial tension test

We can also deduce from previous three figures that in case of the equibiaxial tension test the deviation is maximally 10%, while in other cases the deviations are much larger. Hence, the equibiaxial tension test does not depend very much on the spacing between clamps in case of 50x50mm specimens.

2.2 Influence of specimen dimension

The Camea mechanical testing machine, which was presented in the introduction, has its measuring range of forces up to 200 N. This force limit appears in case of stiff elastomers with a higher thickness and disables then higher values of specimen elongation. One way how to achieve higher elongation values of the specimen can be the use of a smaller specimen. For this purpose, computational simulations of the tests were performed for the specimen with dimensions of 30x30mm and the same dependency between deviation of stress and stretch ratio was determined as in the previous section. Simulations were performed for two different spacing between clamps: b = 12.5mm and b = 15mm. Results are shown in figures 6, 7 and 8.

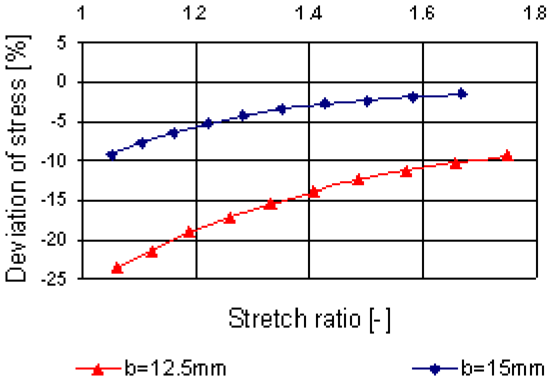

Figure 6 Uniaxial tension test with the specimen 30x30mm

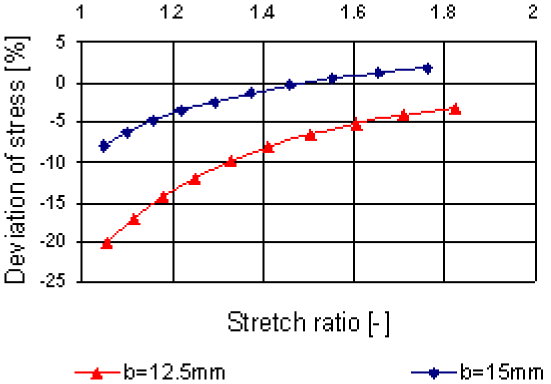

Figure 7 Planar tension test with the specimen 30x30mm

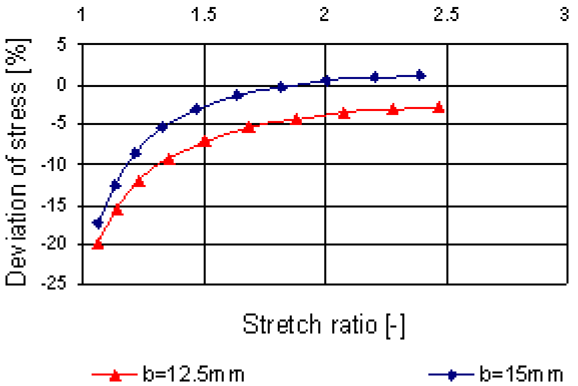

Figure 8 Equibiaxial tension test with the specimen 30x30mm

It is obvious from the previous three figures that the best spacing b = 12.5 mm determined in section 2.1 gives the deviation larger then or equal to 20%. When we have increased the spacing to b = 15mm (the maximal possible spacing), the deviation decreased on cue, but it is still higher than in the previous section. We assumed that a larger spacing between clamps caused smaller elongation of the specimen between clamps, because the clamps are too far from each other. Hence, stress along the length of the specimen is less uniform than in case of the smaller spacing. In order to verify this hypothesis, the computational simulation with smaller specimen 25x25mm and different spacing of clamps were performed. Smaller deviations were expected between stresses than in the previous case of 30x30mm specimen. Results are in figures 9, 10, 11.

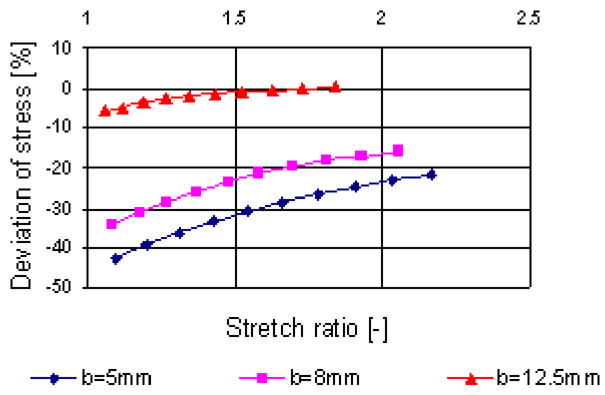

Figure 9 Uniaxial tension test with the specimen 25x25mm

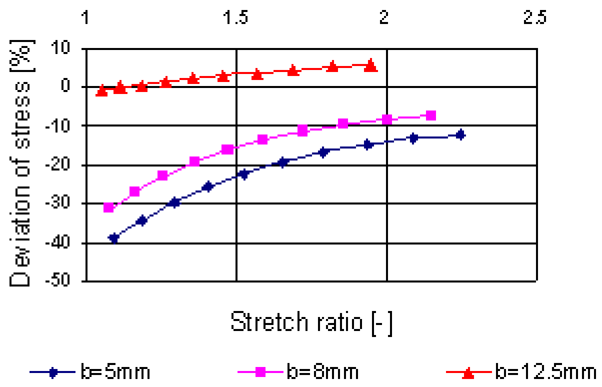

Figure 10 Planar tension test with the specimen 25x25mm

When we compared e.g. figure 7 with figure 10, we have found out that the deviation between stresses in 25x25mm specimen with b = 12.5mm is better than in case of 30x30mm specimen with any spacing; the deviation less than 5% was achieved for the stretch ratio λ approximately 1.25 in the 25x25mm specimen, while in the 30x30mm specimen, the same deviation was achieved for $\lambda > 1.25$. Hence, the use of the smaller specimen with the dimensions of 25x25mm and b = 12.5mm results in a smaller deviation than use of the 30x30mm specimen.

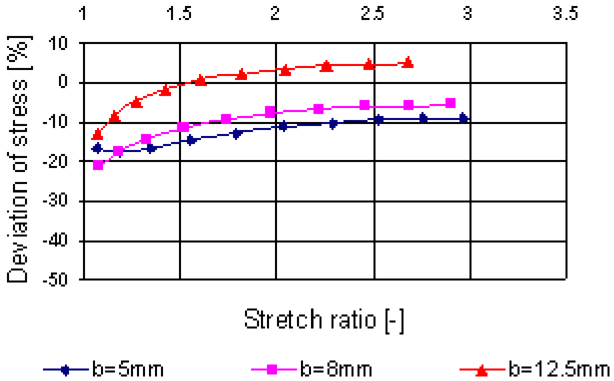

Figure 11 Equibiaxial tension test with the specimen 25x25mm

2.3 Influence of the distance of reference points

The last factor under our study was the distance of the reference points, i.e. the reference distance L0. We consider three different distances: 10%, 40%, and 70% of L1, where L1 is distance between clamps according to figure 12b). In this figure specimens with L0 = 0.1L1 (fig. 12a) and with L0 = 0.7L1 (fig. 12b) are depicted. All the previous simulations were performed with L0 = 0.4L1.

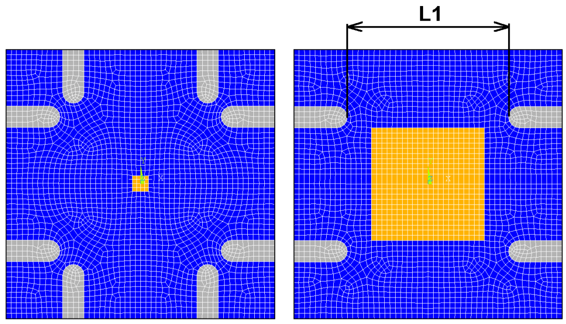

Figure 12 Different distances of the reference points

Results for the specimen 25x25mm and the spacing of b = 12.5mm are presented in figures 13, 14 and 15, and results for the specimen 50x50mm, b = 12.5mm are in figures 16,17 and 18. By analyzing these figures it was found out that the change of the reference distance does not improve the results substantially. The results obtained with L0 = 0.1 L1 or L0 = 0.7L1 are worse or not substantially better than the results obtained with L0 = 0.4L1. In other words, the reference distance L0 = 0.4L1 is an optimal one. An exception of this statement occurs only in case of the equibiaxial tension test with the 25x25mm specimen (fig. 14), because L0 = 0.1L1 results in a much lower deviation for small stretch ratios than in the other cases. However, this exception is worth only when we do not consider the other tests (uniaxial and planar tension tests), otherwise we have obtained a better deviation in the equibiaxial test, but worse deviations in the other tests.

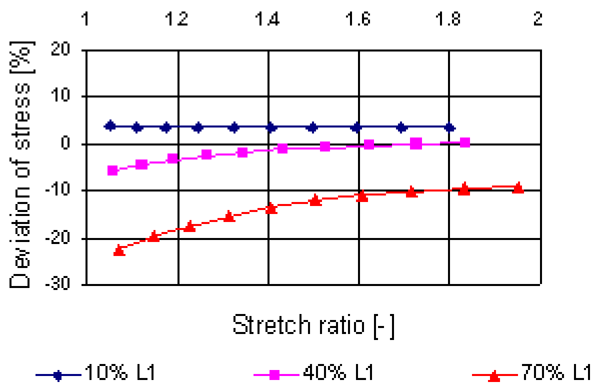

Figure 13 Uniaxial tension test with the specimen 25x25mm

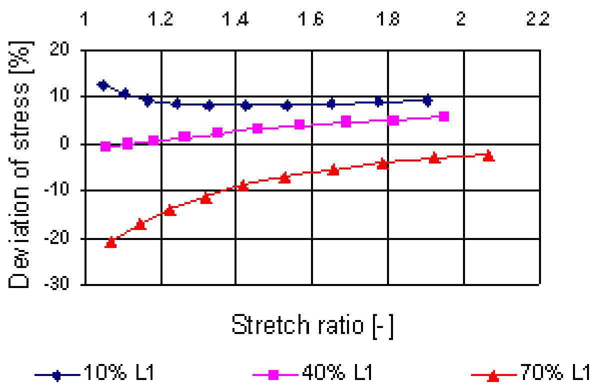

Figure 14 Planar tension test with the specimen 25x25mm

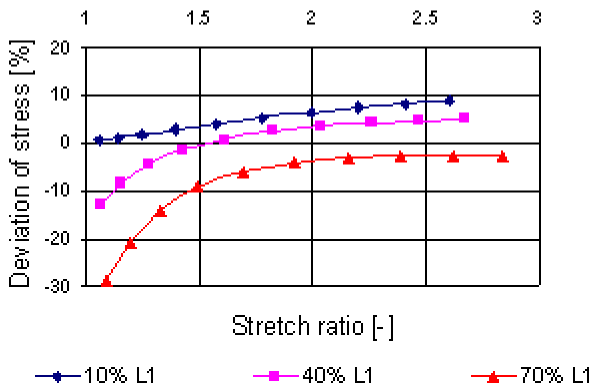

Figure 15 Equibiaxial tension test with the specimen 25x25mm

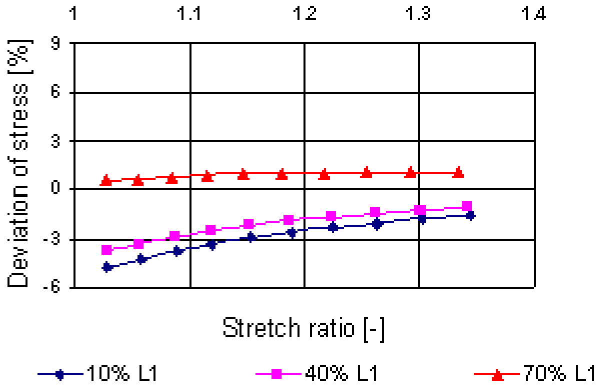

Figure 16 Uniaxial tension test with the specimen 50x50mm

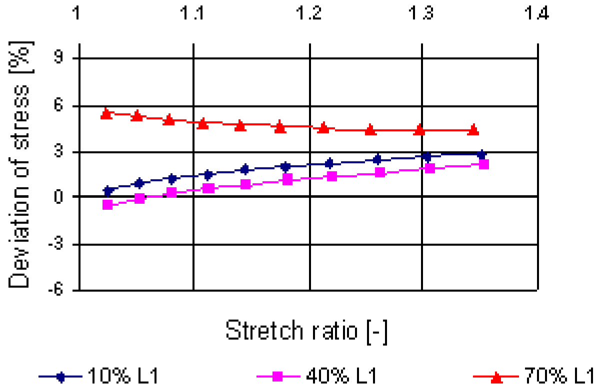

Figure 17 Planar tension test with the specimen 50x50mm

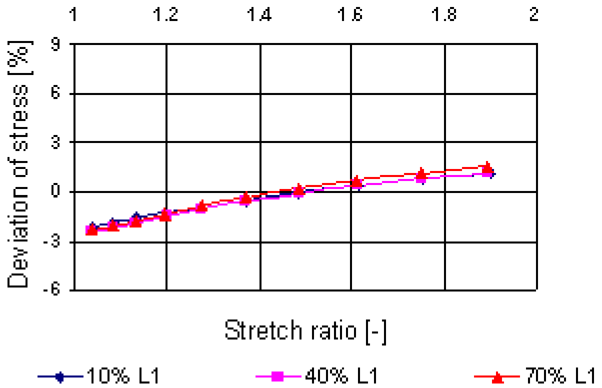

Figure 18 Equibiaxial tension test with the specimen 50x50mm

3. Experiment

Results of experiments depicted in figure 19 have evoked the computational simulations of experiments presented in section 2. This figure refers to the experiments in uniaxial, equibiaxial and planar tension with the specimen 50x50mm with spacing between clamps of b = 8mm. Figure 19 is completed by another uniaxial tension test performed with a specimen with dimensions 20x50mm. It is evident that both uniaxial tension tests differ from each other substantially. Next, the planar test gives nearly the same results like one of the uniaxial tests.

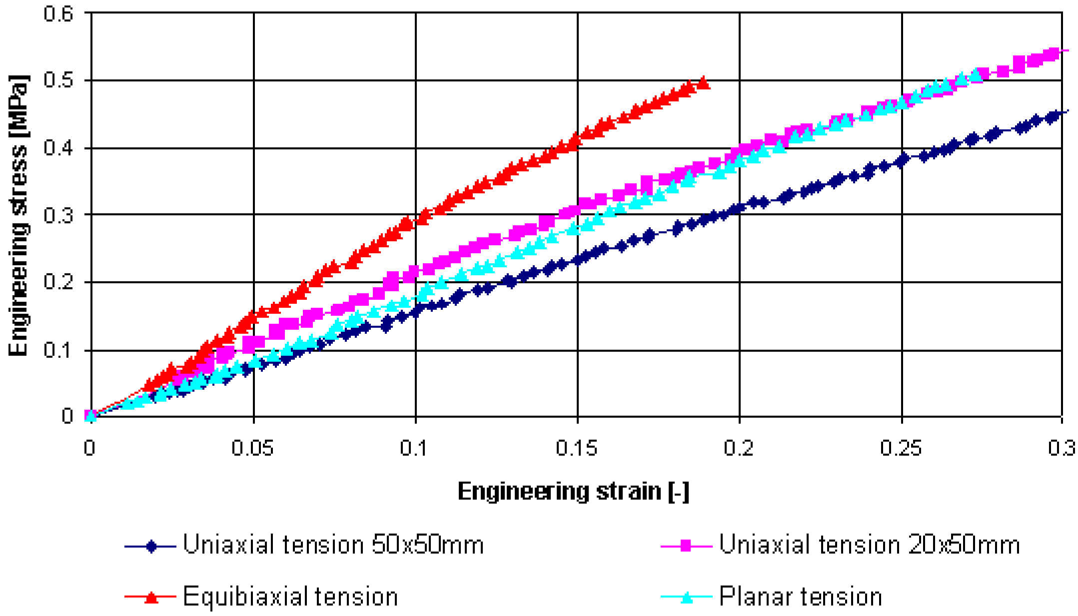

Figure 19 Specimen 50x50mm, spacing b = 8mm

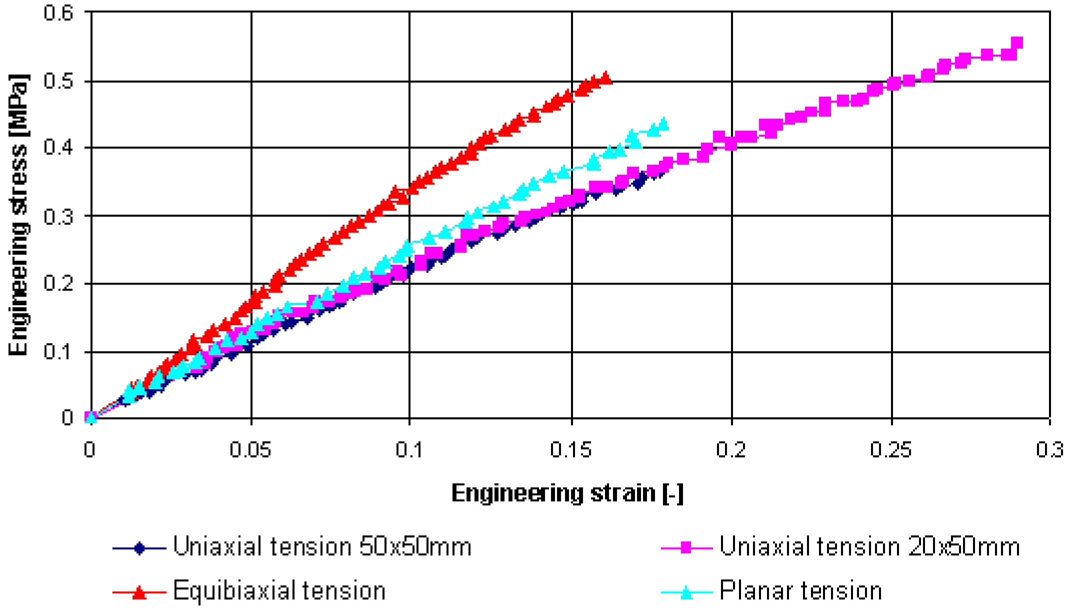

Figure 20 Specimen 50x50mm, spacing b = 12.5mm

The disagreement between both uniaxial tests can be explained on the basis of the results presented in section 2. Cauchy stresses in the narrow specimen are constant all over the width of the specimen, while in case of the wide specimen unseasonable location of clamps caused uneven stresses. In order to found out whether the change in spacing between clamps can cause better agreement between uniaxial tests, new experiments (figure 20) were performed with the same specimen, but with other spacing. The optimal spacing determined in section 2 (b = 12.5mm) was used. Fig. 20 shows that the change of spacing between clamps results in agreement between both uniaxial tension tests, and it also removed the nonlogical agreement between the planar and uniaxial tests. For illustration, the deformed shapes of the specimen under equibiaxial loading are depicted in figure 21, namely in the form obtained both from computational simulation (left specimen) and from experiment (right specimen).

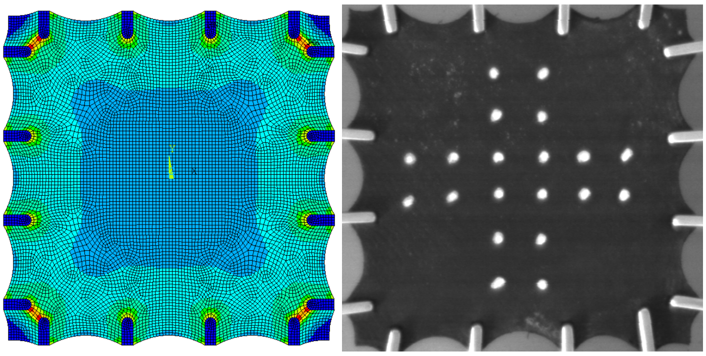

Figure 21 Deformed shapes of the specimen in equibiaxial test (simulation and experiment)

4. Conclusion

The objective of this article was to find out how the three mentioned factors influence the accuracy of the measured stress-strain data. Concerning the influence of the spacing between clamps, it was found out that such a spacing is optimal, which ensures a uniform distribution of stresses and strains over the width of the specimen, i.e. the clamps must be located uniformly over the width of the specimen and not concentrated in its middle part. In case of our specimen with dimensions 50x50mm, the optimal spacing was b = 12.5mm.

Next, it was shown that in case of the smaller specimen it is better to use specimen with dimensions 25x25mm than specimen with 30x30mm. In the smaller one, the optimal spacing distance can be used, which leads to a smaller deviation, because the non-uniformity of stresses (or strains) is not significant. Increase in spacing leads to a larger non-uniformity, because the middle part of the specimen between clamps is much less elongated than the specimen parts under clamps.

The last factor under our study was the distance of the reference points, which are used to determine the specimen deformation. The computational simulations have shown that the change of the reference distance has no substantial effect on improvement of results, and the distance representing 40% of the actual length between clamps (length L1) can be considered as the optimal reference distance.

Generally, the deviation between stresses is minimal at the largest specimen (with 50x50mm dimensions), and the simulations have shown that in case of the equibiaxial test the spacing between clamps has no substantial effect on the accuracy of the stress-strain data.

5. Acknowledgement

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6. References

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