17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

OPTIMAL NUMBER AND LOCATION OF PIEZOCERAMIC ACTUATORS AND SENSORS FOR VIBRATION SUPPRESSION OF TWO-DIMENSIONAL SYSTEMS

P. Benkovský[*], L. Starek[*]

Abstract: *This paper deals with the development of process for finding the optimal number and location of piezoceramic actuators and sensors to suppress the vibration of 2D systems. This choice must satisfy the condition of maximum possible system observability and controllability and is independent of the choice of control algorithm. Controllability and observability of the system is expressed via grammians of controllability and observability, respectively as product of these two items. The problem is solved by the optimization process in MATLAB program using a combination of library functions as well as its own created function and a set of input data from system model from program package ANSYS. Comparing the matrix product of grammians in the optimization process, serves as the main benchmark and its result is a set of indicia location of piezoceramic members.*

Keywords: ***Grammians, 2D systems, optimization, MATLAB, ANSYS.***

1. Introduction

Active vibration suppression is no new concept, but many studies are still concerned with the theoretical description of the problem and are solving problems of specific boundary conditions. The aim of this work is to take advantage of already known theoretical principles to solve more general issues, namely, the systems with non-constant thickness, complex boundary conditions, etc. The main objective is to design optimization algorithm and subroutine that will process the input data. This can expand capabilities of solvers for 1D and 3D systems in future. Creation of a virtual model was realized by FEM in program ANSYS. Next, the output data from ANSYS were transferred to MATLAB for further use. This transfer is realized by set subroutines, which provide all necessary information like mass, stiffness matrix, geometry, indexing of nodes and so on. These subroutines also solve the problem of mixed or missing nodes due matrix reduction. Maximization of observability and controllability grammians was chosen as main condition for this optimization problem. For maintaining simplicity, sensors and actuators are considered collocated. So calculation of grammians can be realized in state-space balanced representation, which guarantees that observability and controllability grammians will be diagonal and the same. Finally, solving the optimization problem for selected modes will return node indexes, if they satisfy the criteria adopted.

In chapter 2 a whole process of solution is described. An example of pinned plate without damping is shown in chapter 2.1.

2. Process of solution

In general, we obtain nodal model from FEM, which will be implemented into our MATLAB solver:

$$M\ddot{q} + Kq = 0 \tag{1}$$

It’s a system without inputs and outputs with n_d degrees of freedom. M, K are mass and stiffness matrices with dimensions (n_d x n_d). q and their second derivatives are displacement vector and acceleration vector with dimensions (n_d x 1). Using prescribed subroutines we will transform this

[*] Ing. Peter Benkovský and prof. Ing. Ladislav Starek, CSc.: Institute applied mechanics and mechatronics, Slovak Technical University in Bratislava, Námestie Slobody 17; 812 31 Bratislava 1; SK, e-mails: peter.benkovsky@ stuba.sk, ladislav.starek@stuba.sk

system to state space and at the same time we will create inputs and outputs of the system (Gawronsky, 2004).

$$x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \boldsymbol{M}^{-1}\boldsymbol{K}x_1 + \boldsymbol{M}^{-1}\boldsymbol{B}_0 u$$

$$y = \boldsymbol{C}_{oq}x_1 + \boldsymbol{C}_{ov}x_2 \tag{2}$$

Where *x* is state vector and a B_0, C_{oq}, C_{ov} are input matrix, output displacement and velocity matrices in a nodal form. *Their dimensions are (2*nd x 1), (nd x s), (r x nd) a (r x nd), where s and r represent the number of inputs and outputs of the system.*

*Now we can calculate obs*ervability and controllability grammians W_o and W_c using Lyapunov equation:

$$AW_c + W_cA^T + BB^T = 0$$

$$A^TW_o + W_oA + C^TC = 0 \tag{3}$$

Using information about balanced representation we can obtain both grammians, that will be diagonal and equal (Starek, 2009, Moore):

$$W_o = W_c = \Gamma$$

$$\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_{2n})$$

$$\gamma_i \geq 0\,, i = 1, \ldots, 2n \tag{4}$$

So now we can define grammians as functions of *i*-th location of inputs and outputs on structure and the degree of observability and controllability for each mode (or mode *m*) by:

$$\Gamma_m = f_m(i) \tag{5}$$

$$M_m = \frac{\Gamma_m(i)}{max(\Gamma_m)} * 100\% \tag{6}$$

Similarly we can define the degree of observability and controllability on the whole structure for all selected modes,

$$S = \frac{\sum Q_m\Gamma_m(i)}{max\sum(\Gamma_m)} * 100\% \tag{7}$$

as summation of functions Γ_m of selected modes in *i*-th location, divided by maximum of this summarized functions. Where Q_m is weighting function for each mode (inspired by Halim and Moheimani, 2003).

2.1. Example

Plate with these attributes has been modelled:

Dimension: 1 x 1 meter

Thickness: 0.01 meter

Material - Aluminum

Density: 2700 kg/m^3

Young modulus: 70 GPa

First three natural frequencies for transverse vibration in ANSYS are:

- 17.828 Hz
- 38.883 Hz (2 symmetric modes)
- 48.261 Hz

Pinned at nodes: 1, 2, 12, and 22 (Fig. 1).

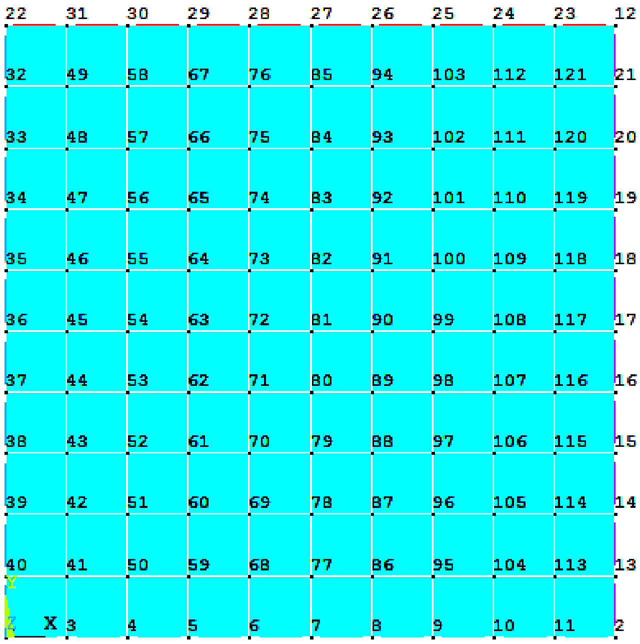

Fig. 1: Plate with map of nodes.

Using created subroutines we will try to find optimal placement for 1, 2 and 4 collocated sensors and actuators for three independent modes.

Tab. 1: Node numbers in dependence of mode and quantity of inputs/outputs.

Number of S/A	Mode 1	Mode 2	Mode 4
1	[81]		
2	[72 81]	[45 117],[7 27]	
4	[72 80 82 90]	[6 8 26 28]	[7 17 36 27]

3. Conclusions

Additional research is needed to verify these results experimentally. The algorithm is working quite well for independent modes, but needs proper weighting function when it is solving a set of modes. Also, with too many combinations, computational time is increasing significantly, which can be remedied by optimizing subroutine for input/output combinations generation. Algorithm was also tested on beam structure with variable thickness over its length, with similar properties. Good solution for independent modes were obtained; and relatively inaccurate results for a set of modes, yet with a better computational time, when the weighting function was not defined properly.

Ackowledgements

The authors gratefully acknowledge the support of the Slovak Ministry of Education Grant Agency VEGA # 1/0744/10.

References

Gawronsky, W. K. (2004) Advanced structural dynamics and active control of structures, Mechanical engineering series, Springer Verlag New York Inc.

Halim, D and Moheimani, S O R (2003) An optimization approach to optimal placement of collocated piezoelectric actuators and sensors on a thin plate, Mechatronics 13 27–47.

Moor, B.C. (1981) Principal Component Analysis in Lienear Systems, Controllability, Observability and Model Reduction, IEEE Transactions on Automatic Control, Vol. 26, pp. 17-32.

Starek, L. (2009) Kmitanie s riadením , Vydala Slovenská technická univerzita v BA v Nakladateľstve STU, Edícia vysokoškolských učebníc, ISBN 978-80-227-3227-7.