17th International Conference
ENGINEERING MECHANICS 2011
Svratka, Czech Republic, 9 – 12 May 2011

ANALYTICAL APPROACH TO CENTRE-LINE OPTIMIZATION OF BURIED ARCH BRIDGES AND ITS BOUNDARIES

M. Foglar*, V. Křístek*

Abstract: *An arch is one of the oldest structural shapes that mankind has invented. The Roman Empire constructed a large number of stone arch bridges on roads or aqueducts, many of which exist to the present. Arch bridges were constructed throughout medieval times; some of them represent masterpieces of the world´s cultural heritage. Buried arch bridges are built on both roads and railways, both as underbridges or overbridges. Due to low construction costs, great durability and endurance, they are favored by contractors and investors. Buried arch bridges are built with spans ranging from 2 m up to 40 m. This paper discusses the structure-soil interaction of buried arch bridges. With the use of an analytical derivation, a method of centre-line optimization of buried arch bridges is proposed and the limitations of this approach discussed.*

Keywords: ***Buried structures, arch, structure-soil interaction.***

1. Introduction

An arch is one of the oldest structural shapes mankind has invented. The Romans constructed a large number of stone arch bridges on roads or aqueducts, many of which exist in present day.

Buried arch bridges are built on both roads and railways, both as underbridges or overbridges. Due to low construction costs, great durability and endurance, they are favored by contractors and investors. Buried arch bridges are built spanning from 2 m up to 40 m.

This paper discusses the structure-soil interaction of buried arch bridges. Utilizing an analytical derivation, a method of centre-line optimization of buried arch bridges is proposed and the limitations of this approach discussed.

2. Loading of buried arch bridges

Buried structures are vertically selfloaded, by impact of the fill, by weight of the fill and in addition due to surface traffic of the buried structure (usually a carriageway). Lateral loading of buried structures is caused mainly by structure-soil interactions, i.e. lateral loading due to compaction of backfill, earth pressure at rest (or active/ passive earth pressure depending on deflection of the structure) and lateral loading due to excess weight.

On frame bridges, the definition of loading is elementary. With buried arch bridges, the load-analysis is complicated by the fact that the horizontal and vertical loads act on the plane of projection of the centre line, see Fig. 1.

An elementary segment of the arch is loaded, according to Fig. 1, by vertical loading *p* /kN/m/ from the fill defined as

$$\mathrm{p} = (\mathrm{h} + \mathrm{y})\,\gamma \tag{1}$$

and lateral loading *s* /kN/m/ defined as

$$s = k\,(h + y)\,\gamma \tag{2}$$

* Ing. Marek Foglar, Ph.D. and prof. Ing. Vladimír Křístek, DrSc.: Department of concrete and masonry structures, Czech Technical University, Thákurova 7; 166 29, Prague; CZ, e-mails: marek.foglar@fsv.cvut.cz, kristek@fsv.cvut.cz

where h /m/ is the height of the fill, γ /kN/m^3/ is the volumetric mass of the fill, y /m/ is the vertical coordinate from the arch crown and k /-/ is the parameter determining the lateral earth pressure. Simplified, the excess vertical and horizontal by traffic can be substituted by increasing the height of the fill.

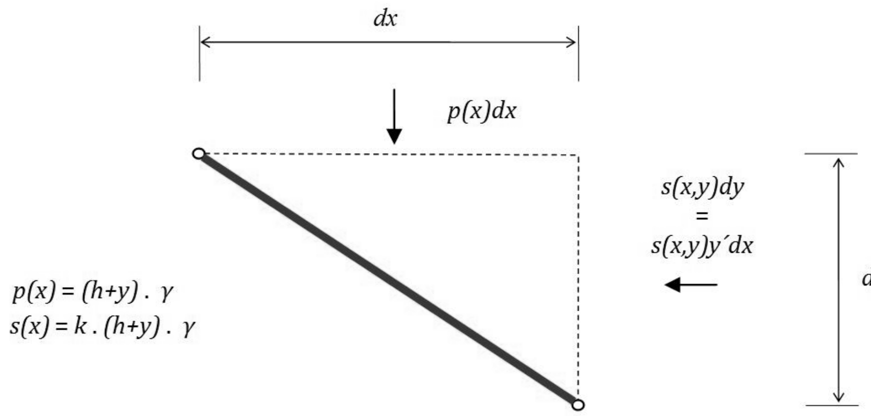

Fig. 1: Loading of an elementary arch segment.

The loading causes deflections of the structure, which changes the initial loading by earth pressure. Fig. 2 shows typical loading and deflection of a buried arch bridge in operation.

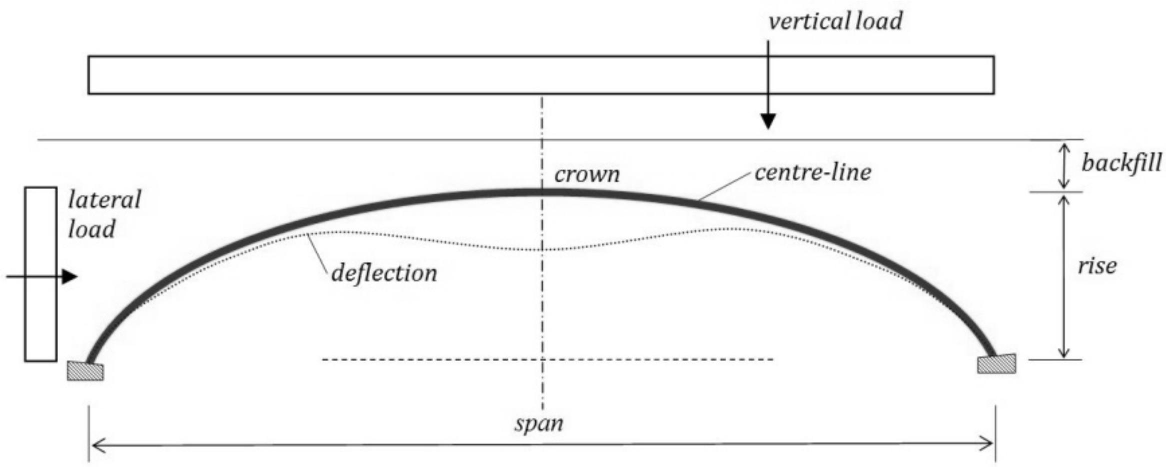

Fig. 2: Buried arch bridge loading.

3. Centre-line optimization of buried arch bridges

The process of centre-line optimization of buried arch bridges leads to a centre-line which is a resultant of forces affecting the structure, thus no perceivable bending moments are present. The optimization results into negligible flexural stresses, therefore almost no bending reinforcement is required thus decreasing the construction time. This topic is discussed in detail in referenced articles.

The shape of the optimized centre-line is defined by a field of loadings affecting the arch. The arch is divided into a finite number of segments; to each of them an interaction vector is attributed. As previously noted and plotted in Fig. 1, the vertical loading is introduced as $p\ (x,y)$, the vertical as $s\ (x,y)$.

The shape of the optimized centre-line $y\ (x)$ is derived on an elementary segment; the axial force in the arch H /kN/ is dissociated into its horizontal part $H(x)$ and vertical part $V(x)$, see Fig. 3.

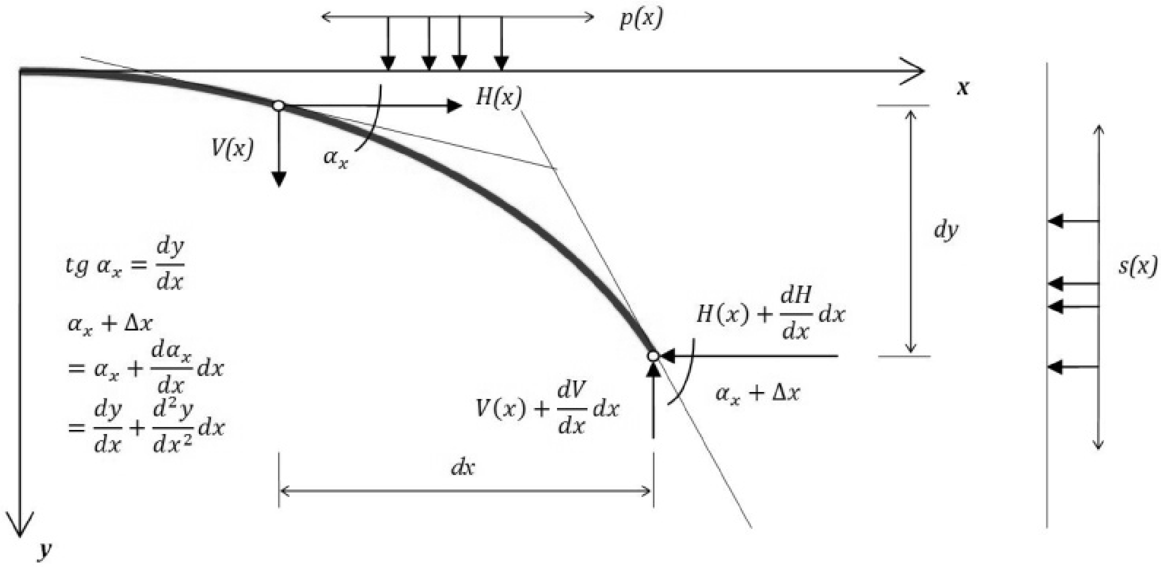

Fig. 3: Derivation of an optimal centre-line.

Horizontal and vertical conditions of equilibrium must be fulfilled:

- horizontally:

$$H(x) - s\,.\,dy - H(x) - \frac{dH}{dx}dx = 0 \tag{3}$$

$$\frac{dH}{dx} = s\,.\,\frac{dy}{dx} \tag{4}$$

$$H' + s\,y' = 0 \tag{5}$$

- vertically:

$$V(x) = H(x)\,.\,tg\,\alpha_x \tag{6}$$

$$H(x)\,.\,tg\,\alpha_x + p\,.\,dx - \left(H(x) + \frac{dH}{dx}dx\right).\,tg\,\alpha_x = 0 \tag{7}$$

The angle α_x is substituted by the expression noted in Fig. 2, the product

$$-\frac{dH}{dx}dx\,.\,\frac{d^2y}{dx^2}\,.\,dx$$

is neglected:

$$H(x)\,.\,\frac{d^2y}{dx^2} + \frac{dH}{dx}.\frac{dy}{dx} - p = 0 \tag{7}$$

$$H\,y'' + H'\,y' - p = 0 \tag{8}$$

Substituting (5) to (8):

$$H\,y'' + s\,(y')^2 - p = 0 \tag{9}$$

Introducing $y' = z\,(x)$:

$$H\,z' + s\,z^2 = p \tag{10}$$

Substituting (1) to (8) and (2) to (5):

$$H\,y'' + H'\,y' - \gamma\,y = h\,\gamma \tag{11}$$

$$H' + k\,\gamma\,y\,y' + k\,h\,\gamma\,y' = 0 \tag{12}$$

The Equations (11) and (12) can be solved in a closed form only under special boundary conditions; therefore a numerical solution is necessary in most cases. The most suitable method for the solution is the pre process of the vector synthesis. For the process of vector creation, the centre line is divided into equal horizontal segments. For each segment, the interaction vector is defined. The numerical application of vector synthesis is fast and robust for all possible loading arrangements.

The method was applied on existing bridges (Foglar et al. 2010), great reductions of bending moments were achieved, see Fig. 4. Though great reductions (from 1400 kNm to less than 200 kNm at the restraint), the ideal state, i.e. total reduction of bending moments, was not achieved.

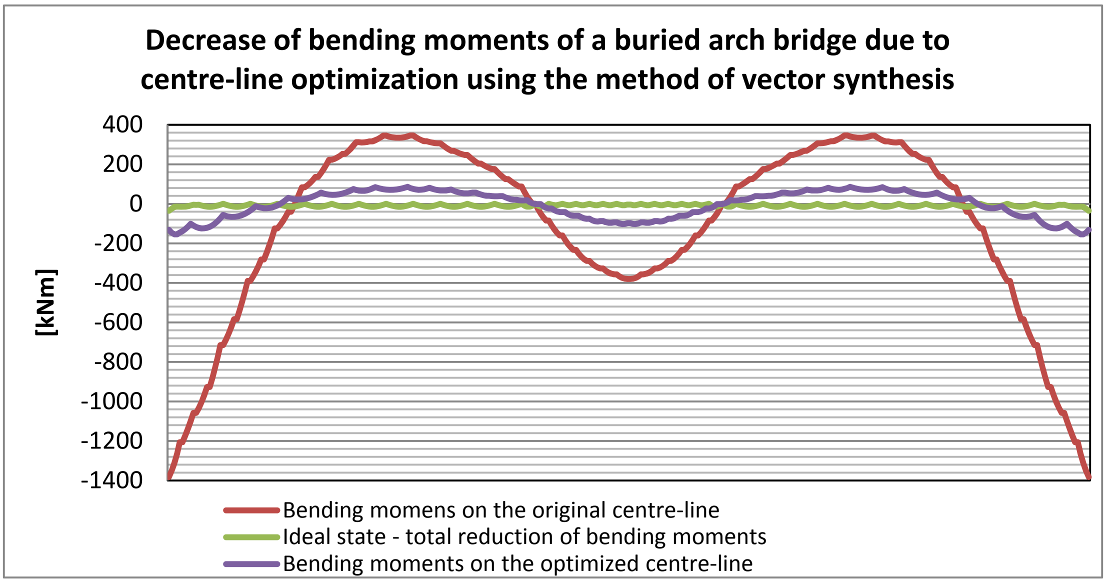

Fig. 4: Reduction of bending moments due to centre-line optimization.

This problem is attributed to the slip between soil and the structure. In FEM modeling, the slip is modeled by contact elements with a friction coefficient, whose value is derived empirically. The analytical approach must be modified to incorporate this phenomenon.

4. Conclusions

This paper discussed the structure-soil interaction of buried arch bridges. With the basis of an analytical derivation, a method of centre-line optimization of buried arch bridges was described and its boundaries discussed. Further research is outlined.

Acknowledgement

This text presents outcomes of the projects supported by the Czech Science Foundation, project No. 103/09/2071 and Czech Ministry of Education project MSM 6840770005.

References

Foglar, M. & al. (2010) The use of fiber reinforced concrete in buried arch bridges (in Czech). Beton TKS, 10, 2, pp. 92-95.

Foglar, M. & Kristek, V. (2010) Structure-soil interaction of buried arch bridges, in: Frydrysek & Nikodym (ed.), Beams and frames on elastic foundation 3, Ostrava – VSB – Technical University of Ostrava, Ostrava, pp. 303-319.