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COMPARISON OF NONLOCAL AND GRADIENT-ENHANCED DAMAGE-PLASTICITY MODEL

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Abstract: *This paper deals with a damage-plasticity model combining Mises plasticity with isotropic damage. Due to softening induced by damage growth, the governing differential equations lose ellipticity, which leads to an ill-posed boundary value problem. From the numerical point of view, ill-posedness is manifested by pathological sensitivity of the results to the size of finite elements. To avoid this undesired behavior, the model is regularized by a nonlocal integral formulation or, alternatively, by an implicit gradient formulation. The difference between the formulations is discussed and the behavior of both regularized models is illustrated by a numerical example.*

Keywords: ***Nonlocal continua, damage-plasticity model, finite elements.***

1. Introduction

This paper presents a coupled damage-plasticity model. Continuum damage mechanics is suitable for the description of stiffness degradation due to the growth of defects such as micro-voids and micro-cracks, while plasticity theory describes permanent deformations of a material induced e.g. by slip mechanisms. However, standard damage-plasticity models with softening would lead to physically meaningless results. In this contribution, two methods that can provide an objective description of localized inelastic processes are described. Both can be classified as strongly nonlocal (Bažant and Jirásek, (2002)) and introduce a new material parameter with the dimension of length.

2. Constitutive model

In this section, the constitutive model combining the classical Mises plasticity with isotropic damage is described. We restrict our attention to the small strain theory, thus the kinematic framework is based on the additive decomposition of strain into the elastic part and the plastic part,

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_p \,. \tag{1}$$

The stress-strain law has the form

$$\boldsymbol{\sigma} = (1-\omega)\bar{\boldsymbol{\sigma}} = (1-\omega)\mathbf{D} : \boldsymbol{\varepsilon}_e \,, \tag{2}$$

where $\boldsymbol{\sigma}$ is the nominal stress tensor, $\bar{\boldsymbol{\sigma}}$ is the effective stress tenor, $\mathbf{D}$ is the fourth-order stiffness tensor and ω denotes a scalar damage variable. The yield condition is formulated in terms of the effective stress, and so the Mises yield function is written as

$$f(\bar{\boldsymbol{\sigma}},\kappa) = \sqrt{\frac{3}{2}\bar{\mathbf{s}} : \bar{\mathbf{s}}} - \sigma_Y(\kappa)\,, \tag{3}$$

where $\bar{\mathbf{s}}$ is the deviatoric part of effective stress.

The loading-unloading conditions are expressed in the usual form

$$f(\bar{\boldsymbol{\sigma}},\kappa) \le 0 \qquad \dot{\lambda} \ge 0 \qquad \dot{\lambda} f(\bar{\boldsymbol{\sigma}},\kappa) = 0\,. \tag{4}$$

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The yield function (3) incorporates the isotropic linear hardening rule

$$\sigma_Y(\kappa) = \sigma_0 + H\kappa\,. \tag{5}$$

where H denotes the hardening modulus and κ is the cumulated plastic strain defined by the evolution law

$$\dot{\kappa} = \sqrt{\dot{\varepsilon}_p : \dot{\varepsilon}_p}\,. \tag{6}$$

The associative flow rule is considered,

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\lambda}\frac{\partial f(\bar{\boldsymbol{\sigma}},\kappa)}{\partial \bar{\boldsymbol{\sigma}}}\,. \tag{7}$$

The growth of damage is driven by the cumulated plastic strain and is described by the exponential law

$$\omega(\kappa) = \omega_c\left(1 - \mathrm{e}^{-a\kappa}\right). \tag{8}$$

In the equation above, ω_c is the critical damage and a is a positive dimensionless parameter that controls the softening part of the stress-strain diagram.

3. Regularization techniques

The local model cannot be used after loss of ellipticity of the governing equation, because of the resulting pathological sensitivity of the numerical results with respect to the size and orientation of the finite element mesh. The present model is regularized by the integral or gradient formulation. In both approaches, the damage variable is computed from the over-nonlocal cumulated plastic strain while the plastic part of the model remains local. The over-nonlocal cumulated plastic strain is computed as

$$\hat{\kappa} = (1-m)\kappa + m\bar{\kappa}\,, \tag{9}$$

where $\bar{\kappa}$ is the nonlocal cumulated plastic strain and m is a model parameter. Full regularization can be achieved only if parameter m is greater than 1 (Jirásek and Rolshoven, 2003). Evaluation of the nonlocal cumulated plastic strain follows a different procedure for the integral formulation and for the gradient formulation.

3.1. Integral-type nonlocal formulation

The integral nonlocal formulation is based on weighted spatial averaging of the cumulated plastic strain:

$$\bar{\kappa}(x) = \int_V \alpha(x,s)\kappa(s)ds\,, \tag{10}$$

with the nonlocal weight function $\alpha(x,s)$ defined as

$$\alpha(x,s) = \frac{\alpha_0(\|x-s\|)}{\int_V \alpha_0(\|x-t\|)dt}\,. \tag{11}$$

Here,

$$\alpha_0(r) = \left(1 - \frac{r^2}{R^2}\right)^2. \tag{12}$$

is a monotonically decreasing function for $r < R$ and vanishes if the distance r exceeds the nonlocal interaction radius R.

3.2. Gradient-enhanced formulation

The gradient formulation can be conceived as the differential counterpart to the integral formulation. Here we focus on the implicit gradient formulation, which is more robust than the explicit one and requires only C^0 continuous finite element approximation. Instead of evaluation of the integral in (10), the nonlocal cumulated plastic strain is computed from a Helmholtz-type differential equation

$$\bar{\kappa} - l^2 \nabla^2 \bar{\kappa} = \kappa \,, \tag{13}$$

with homogeneous Neumann boundary condition. In (13), l is the length scale parameter and ∇^2 is the Laplace operator. It can be shown that the implicit gradient formulation is equivalent to the integral one with a special weight function, but the numerical implementation is quite different.

4. Numerical implementation

The numerical implementation of the local version of the model consists of the classical radial return algorithm followed by an explicit evaluation of damage. In the integral theory, nonlocal cumulated plastic strain is evaluated from the integral (10), which is in the numerical implementation replaced by the finite sum

$$\bar{\kappa}_k = \sum_{l=1}^{N_{GP}} w_l J_l \alpha_{kl} \kappa_l \,, \tag{14}$$

where w_l is the integration weight, J_l is the Jacobian of the isoparametric transformation, and

$$\alpha_{kl} = \frac{\alpha_0\left(\left\| x_k - x_l \right\|\right)}{\sum_{n=1}^{N_{GP}} w_n J_n \alpha_0\left(\left\| x_k - x_n \right\|\right)} \tag{15}$$

is the weight of interaction between points k and l. α_{kl} vanishes if the distance between points k and l is larger than the nonlocal integration radius R, so the sum can be evaluated only for Gauss points l inside the circle radius R centered at Gauss point k.

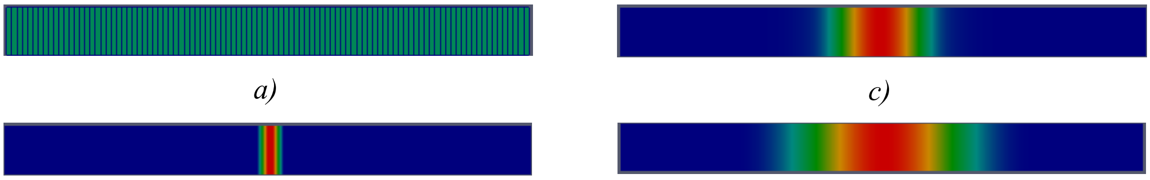
Implementation of the implicit gradient approach leads to a coupled problem. The Helmholz equation (13) is coupled with the standard equilibrium equation, and new degrees of freedom (which correspond to nodal values of the nonlocal variable) are introduced. It is shown in (Simone et al., 2003) that using the same order for the interpolation functions for the displacement and for the nonlocal cumulated plastic strain could lead to spurious oscillations in the stress field. To suppress such oscillations, a quadratic approximation of displacement and a linear approximation of nonlocal cumulated plastic strain are used in the simulation.

One advantage of the integral formulation is that the kinematic and equilibrium equations remain standard and the strain and stress keep their usual form, but the computational cost is increasing since the profile of nonzero elements of the nonlocal consistent algorithmic tangent stiffness is growing during the simulation due to the nonlocal interaction between Gauss points; see Jirásek & Patzák (2004) and (Horák et al., 2011) for derivation and other numerical aspects of the consistent nonlocal algorithmic tangent stiffness. The main disadvantage of the implicit formulation consists in the introduction of new nonlocal degrees of freedom with non-straightforward physical meaning; on the other hand the computational cost does not depend on the internal length.

5. Numerical simulation

Both nonlocal models were implemented into OOFEM, an object-oriented finite element code (Patzák, & Bittnar, 2001; Patzák, Rypl & Bittnar, 2001). To show that the models remove pathological sensitivity to the size of finite elements, a simple uniaxial tensile test of the bar clamped at one end and loaded by a prescribed displacement incrementally applied at the free end is simulated. The yield stress in the central part of the bar is reduced by 10% to trigger localization. The following geometrical and material parameters were selected: bar length $L = 100$ cm, cross-sectional area

$A = 1$ cm^2, Young's modulus $E = 40\ 000$ MPa, initial yield stress $\sigma_Y = 100$ MPa, hardening modulus $H = 0$ MPa, damage parameters $\omega_C = 0.7$, $a = 60$ and nonlocal parameter $m = 1.5$, internal length for nonlocal model $R = 2$ cm and internal length for gradient model $l = 2$ cm. The finite element mesh and the development of damage along the bar are plotted in Fig. 1. The size of localization zone for the local version of the model is dictated by the element size. The gradient and integral models lead to meaningful results with a localization band extending over several elements. The width of the band depends on the choice of the length scale parameters.



a) *c)*

b) *d)*

Fig. 1: a) Finite element mesh, b) damage distribution for the local model, c) damage distribution for the gradient formulation, d) damage distribution for the integral formulation.

6. Conclusions

We have presented a model that combines Mises plasticity with isotropic damage mechanics, and introduced two remedies which lead to an objective description of localized failure processes. Further research will focus on the comparison of the computational efficiency of both models in two-dimensional and three-dimensional large-scale problems, and on extensions of the gradient regularization to more general yield conditions.

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