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ADDED MASS AND DAMPING OF INCOMPRESSIBLE VISCOUS FLUID

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Abstract: *Approach for determination of the dynamic behavior of an elastic body submerged in fluid is presented in this contribution. Method is based on independent solution of the fluid and structure. Commercial computational fluid dynamic software (ANSYS CFX) is used for determination of added effects of the fluid. These added effects are included into finite element model of the body in form of MATRIX27 finite element.*

Keywords: ***Fluid structure interaction, FEM, added mass, added damping, ANSYS.***

1. Introduction

An incompressible fluid which surrounds an elastic body has significant influence on his dynamic behaviors. If the body is displaced, surrounding fluid is also displaced to accommodate its moving. This move generates pressure change and forces acting on this solid body. This effect has significant influence on eigenfrequencies and damping value. Computation of these values is important for effective construction of a hydraulic machine, vessels and so on.

Researches have been interested in this problem for long time and lots of various approaches have been used for its solution. An added mass which replace the inertia effects of the fluid can be solved. Another possible approach is direct solution of eigenfrequencies of the fluid-structure system. Simulation in time domain and subsequent evaluation of the response can also be used.

Finite element method (FEM) is widespread use because it allows solution of body which has general shape. Pressure field in the compressible fluid as solution of the wave equation has been presented by Zienkiewicz. FEM is used for determination of the displacement of the structure and also pressure in the fluid. This description gives one system of linear equations. Solution of the eigenvalue problem of the nonsymmetrical matrices gives eigen values and eigen vectors. Similar approach has been described in Schroeder (1975). Laplace equation for description of the incompressible fluid is used for example by Altinisik (1981) and transformation of the matrices to a symmetric form was presented by Everstine (1981).

More advanced than previous approach is using of potential flow. FEM also determines displacement of the structure and flow in the fluid. This method can be used for determination of the eigenfrequencies an eigenmodes for the structures in flowing fluid. Nitikitpaiboon (1993) introduce ALE (Arbitrary Lagrangian-Eulerian) description for this approach.

Solution in time domain is most universal. Monolithic approach has been used by Zhang (2003). The structure deformation and fluid flow is solved in one system of the equation. Main advantage of the partitioned approach is that widespread commercial computer programs can be used. On the other hand, solution in time domain can be complicated because ALE which is used for representation of the displacement of the fluid boundary is not suitable for problems with small deformations. Combination of the Lagrangian desription for the structure and Arbitrary Lagrengiane Eulerien desription for the fluid flow on the moving grid my cause problems with numerical stabilyty. Transpiration method tries to remove this disadvantage. This method solves the problem on fixed grid and special transpiration

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boundary condition is used. de Morais (2007) compared results of this method with results which was obtained by using ALE.

2. Methods

Inclusion of the influence of fluid on the elastic body in the form of added effects (added mass, added damping, added stiffness) is simplest and most effective. In the case of viscous fluid, this approach is not exact. Exact solution of the influence of viscous fluid on the body is very difficult and may not be feasible. If the fluid is incompressible viscous and has no mean flow can be body motion expressed in simplified form:

$$(\boldsymbol{m_{add}} + \boldsymbol{m})\ddot{\boldsymbol{x}} + (\boldsymbol{b_{add}} + \boldsymbol{b})\dot{\boldsymbol{x}} + \boldsymbol{kx} = \boldsymbol{F_{ext}} \tag{1}$$

Where: $\mathbf{m_{add}}$ - added mass matrix, $\mathbf{m}$ - mass matrix, $\ddot{\mathbf{x}}$ - acceleration vector, $\mathbf{b_{add}}$ - added damping matrix, $\mathbf{b}$ - damping matrix, $\dot{\mathbf{x}}$ - velocity vector, $\mathbf{k}$ - stiffnes matrix, $\mathbf{F_{ext}}$ - external forces matrix.

Some simplifying assumptions must be accepted to obtain additional effects. Neglect of convective term of the Navier-Stokes equation or irrotational flow may not be suitable for this task. Determination of the fluid force act on the body can be difficult in this case. Determination of damping value by linearized model has beem valid only for extremely small displacement, but displacement has been much greater in practice.

Approach which is presented is based on independent solution of deformation of the elastic body and flow in the fluid (Pochylý, 2008). The solution assumes that solid body vibrates harmonically and modal shape of the isolated body is the same as modal shape of the body which is surrounded by fluid. This approach is significantly modified. The neglection of the convective term of the N-S equation and Laplace transformation are not used for solution. The motion of the elastic body is described by equation (1). The flow in the fluid is expressed by simplyfied Navier-Stokes equation in form (2) and continuity (3). This system of equations is linear.

$$\frac{\partial \boldsymbol{c}}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial \boldsymbol{x}} \tag{2}$$

$$\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}} = 0 \tag{3}$$

Where: $\mathbf{c}$ – velocity vector, t – time, p – preassure, $\mathbf{x}$ – coordinate, $\mathbf{w}$ – modal shape, $\mathbf{n}$ – normal vector. Boundary condition on the interface is:

$$\mathbf{c} = \ddot{x}_s \boldsymbol{w} \tag{4}$$

Added mass is obtained as force which fluid acts on the body, which is moved by unit acceleration (5). Assumption is that displacements of the body are small given the size of the body and fluid velocity is also small.

$$\boldsymbol{m}_{add}\ddot{x}_{\boldsymbol{s}}\boldsymbol{w} = \int_S p\,\boldsymbol{n} dS \tag{5}$$

Commercial computational programs ANSYS and CFX are used for solution. Solution must be provided in time domain so that fluid velocity on the interface is increased during time steps. Resulting velocity on the end of the solution must be small. Full Navier-stokes equations are used in CFX, but above described procedure cause that are significant only terms playing in equation (2-4). Calculated added mass is crucial for the eigenfrequency value.

Added damping is similarly obtained from force which fluid acts on the body if this is moved by constant velocity. Small displacement of the body is assumed. The flow in the fluid is expressed by Navier-Stokes equations (6) and continuity (7). Navier-Stokes equation haw some term neglected and ALE description is used. This system of equations is not linear, but we will treat it as a linear.

$$(\boldsymbol{c} - \dot{\boldsymbol{v}})\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}} = -\frac{1}{\rho}\frac{\partial p}{\partial \boldsymbol{x}} + \frac{\mu}{\rho}\frac{\partial^2 \boldsymbol{c}}{\partial \boldsymbol{x}^2} \tag{6}$$

$$\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}} = 0 \tag{7}$$

Boundary condition on the interface is:

$$\dot{\boldsymbol{v}} = \dot{x}_S \boldsymbol{w} \tag{8}$$

Procedure of solution of the added damping in CFX is similar as in the previous case.

$$\boldsymbol{b}_{add}\dot{x}_s\boldsymbol{w} = \int_s \; p\,\boldsymbol{n}dS + \mu \int_S \; \frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}} ds \tag{9}$$

Where: $\dot{\mathbf{v}}$ – mesh velocity, μ – dynamic viscosity, $\dot{x}_s$ – selected velocity, $\ddot{x}_s$ – selected acceleration.

Solved added damping then will be correct only for corresponding frquency and amplitude of the vibration.

Thus, obtained value of the added mass and added damping can be used for solution of the dynamic behavior of the body in ANSYS.

3. Numerical example

The procedure is used for determination of the first eigenfrequency of turbine runner. Modal shape of the isolated runner is obtained by using ANSYS. This shape is used for determination of the added mass by using CFX. Fig. 1 shows pressure field on the face of the runner which is moved by unit acceleration. Fluid force act on the individual nodes of the computational grid gives added mass appropriate of this node. Added mass is inserted into node in form of MATRIX27 finite element. New finite element model with added mass determine eigenfrequency appropriate of this shape. This eigenfrequency can be used for approximate calculation of the damping for selected amplitude.

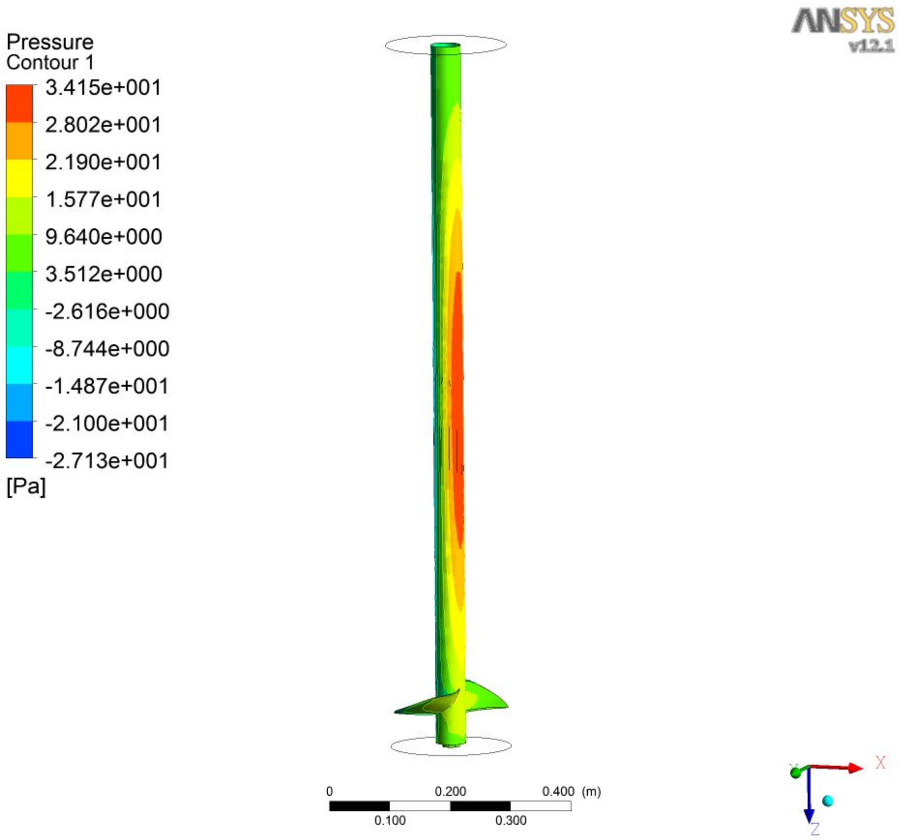

Fig. 1: Pressure field on the face of the runner.

Effective velocity $\dot{x}_s$ is calculated from the eigenfrequency and selected value of amplitude. Added damping is obtained for this velocity and is also inserted into FEM model in form of MATRIX27 finite element. This new model is useable only for transient simulations.

Resulting lowest eigenfrequency obtained by this method is 75.82 Hz. This value is in good agreement with experiment. Experimentally measured value is 74.1 Hz (Malenovský et al., 2010).

4. Conclusion

Above described solution is not exact and is usable only in case of stagnant fluid. Obtaining of the exact solution of the dynamic behavior is very difficult in this case. Real modal shape of the body surrounded by fluid may be different from modal shape of the isolated body. Computed value of the damping must be compared with experiment because real action of the fluid on the body is more complicated than this model assume. On the other hand, this approach seems to be suitable replacement of the complicated solutions in time domain.

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