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STRUCTURAL AND MATERIAL ANALYSIS OF CERAMIC MATRIX COMPOSITES

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Abstract: *This paper deals with the structural and material analysis of ceramic matrix composites. Especially composite systems with plain weave textile reinforcement, where attention is paid to the design of an idealized geometric model based on a statistically equivalent periodic unit cell and the verification of homogenization procedures used for the determination of effective elastic properties, which were solved by the Mori-Tanaka scheme and the finite element method. The results were then compared with experimental measurements.*

Keywords: ***Textile composite, homogenization, Mori-Tanaka scheme, finite element method.***

1. Introduction

Ceramics shows a significant brittleness which is their main disadvantage. One of the possibilities for the improvement of the resilience is adding of the textile reinforcement. Common textile reinforcements are glass, ceramic, carbon or basalt fibers.

Composite materials are increasingly used in various branches. With their physical and chemical properties, composite materials overcome application possibilities of the most commonly used materials, such as metals, concrete and other materials and allow for creation of products with entirely unique properties.

In practice a composite material is mostly manufactured in one step with a final product and therefore it is difficult to separate properties of the material and utility properties of the final product. That is why there exists a very limited database of construction dates for composites themselves. It is the reason for using mathematical models in technical practice - simulations enabling prediction of properties of the designed composite systems based on the knowledge of properties of reinforcing fibers and polymer matrices, the knowledge of time-temperature modes of matrices curing etc. Numerical evaluation of effective properties also requires information about complex inner geometry of the composite system, which depends on a technology of composite fabrication and on a type of applied textile reinforcement.

Nowadays, composites with a textile reinforcement are one of the most dynamically developing group of "new materials". They are among heterogeneous materials with a complex, mostly porous structure. Even at microstructure level it cannot called homogeneous. Modeling of such material is very demanding and therefore the significant simplification is introduced. If the difference between the values obtained by the modeling and values obtained experimentally is acceptable, the prediction of results of computer simulations becomes a perfect tool for saving time and funds in the evaluation of effective properties (Tomková, 2006).

This paper is focused on the conception of geometrical model based on the statistically equivalent periodic unit cell of the analyzed samples. Furthermore, effective elastic properties on these cells using the Mori-Tanaka scheme and the finite element method are calculated.

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2. Experiment

The material properties of individual phases and the knowledge of inner structure are indispensable for the evaluation of effective properties of composites. Therefore, the different composite specimens were fabricated within the framework of our experimental study and subsequently cut to obtain their cross-sections. After which were then grinded, polished and finally scanned.

Because of the fabrication difficulty of ceramic composites, we have focused on the composites with a polymer matrix in the development of geometric model and testing of homogenized procedures. Therefore, measuring of elastic properties has also been done on these composites. All specimens were cured at 200°C. Because of the assumption that a geometry of composite with a ceramic matrix almost does not change in comparison with a polymer matrix composite, at least from the perspective of porosity, the most influencing phase significantly affecting the final effective properties, the polymer matrix composite is sufficient for the mechanical modeling and the development of geometric model. This fact comes from the earlier experimental temperature influence research, see (Šejnoha et al., 2012).

2.1. Experimental evaluation of effective elastic properties

Young's modulus of elasticity and shear modulus of materials have been experimentally determined by ultrasonic method. It is a non-destructive method based on a measurement of speed of waves spreading in three different planes of measured body immersed in water. In this way all elements of the stiffness matrix are determined. Therefore, this method is a very convenient tool for the determination of effective properties. More detailed description of this method is available in (Vorel et al., 2013).

Measurements have been carried out on four samples, two composites with basalt and two with carbon textile reinforcement. One measurement for each mentioned composite has been done on sample along the warp direction and the other measurement on sample along the weft direction. Experimental measurements have been obtained in laboratory LCTS, Université de Bordeaux, 3 allée de la Boétie, 33600 Pessac, France. These are summarized in Table 1.

Tab. 1: Experimental measurements

material - cross-section	E_{11}	E_{22}	E_{33}	G_{12}	G_{13}	G_{23}
	[GPa]					
basalt - warp	*16,3*	*16,3*	*3,8*	*11,5*	*1,7*	*1,7*
basalt - weft	*7,9*	*6,9*	*3,2*	*11,9*	*1,8*	*1,6*
carbon - warp	*42,2*	*41,6*	*3,5*	*35,9*	*0,7*	*0,7*
carbon - weft	*41,6*	*47,7*	*3,5*	*32,5*	*1,2*	*1,2*

In Table 1 axis 1 corresponds to the direction of warp, axis 2 to the direction of weft and axis 3 is in the direction of the composite thickness.

In the numerical determination of effective elastic properties, the distinction between warp and weft directions is not considered, thus assuming in-plane isotropy.

3. Numerical evaluation of effective elastic properties

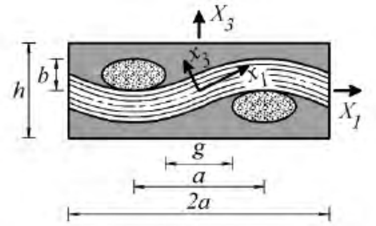
3.1. Input parameters for calculation

Evaluation of effective elastic properties using the Mori-Tanaka and the finite element methods requires information about volume fraction and material parameters of individual phases. The knowledge of geometry is also needed in the case of finite element method.

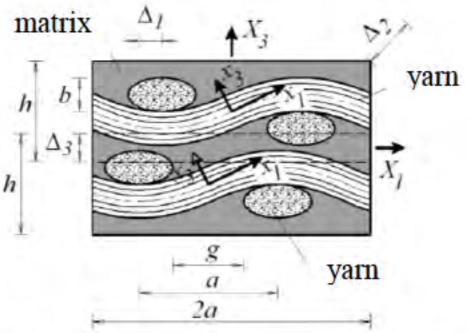
3.1.1. Geometry determination

If information about real microstructure is not reduced only to the knowledge of the volume fraction when determining the effective properties, homogenization principle consists in evaluation of appropriate representative volume element of the material - (RVE) assuming periodic microstructure, for more see (Hatta and Taya, 1986). RVE that coincides with the smallest periodically repeated cell is called the periodic unit cell - PUC. In case of irregular composite microstructure the real microstructure is optimized by the statistically equivalent periodic unit cell - (SEPUC), see (Šejnoha, 2007). For the detailed description on the evaluation of RVE see (Zeman, 2003), for definition see e.g. (Hill, 1963).

In order to acquire geometric parameters of SEPUC (Figure 1) the optimization procedure has been performed for the cross-sections of composites in the binary form. An example of the transformation of a scanned image into binary form is displayed in Figure 2. In the geometry determination porosity does not take any part, therefore, pores in the binary images merge with the matrix and so porosity can not affect the obtained geometry. Porosity is considered later when calculating the effective properties. The results from optimizations are summarized in Table 2.



a) Idealized geometric model of single ply SEPUC



b) Idealized geometric model of two layer SEPUC
Fig. 1: Geometric model of SEPUC

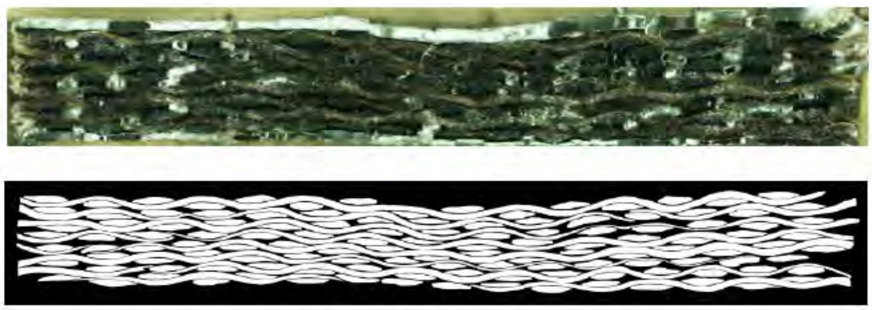


Fig. 2: An original and binary image of the composite with basalt reinforcement

Tab. 2: Geometric parameters of SEPUC

Geom. parameter [µm]	Reinforcement material		
	glass	*basalt*	*carbon*
a	*2121,57*	*862,711*	*2035,58*
b	*122,436*	*87,0123*	*146,828*
g	*889,848*	*311,755*	*464,855*
h	*257,801*	*183,197*	*313,508*
Δ_1	*1,824727*	*416,822*	*14,2212*
Δ_2	*1,824727*	*416,822*	*14,2212*
Δ_3	*-63,6581*	*-52,6671*	*-63,0365*

Parameters Δ_1, Δ_2 and Δ_3 represent translations of yarns in the directions of axis X_1, X_2 and X_3, as it is displayed in Figure 1. In case of $\Delta_3 = 0$ the axial distance of these fibers in the direction of X_3-axis would be equal to the parameter *b*.

3.1.2. Volume fraction determination of particular phases

Volume fractions of particular phases (yarn, matrix, macroscopic pores) were determined from 2D images by means of the image processing toolbox included in MATLAB software, see Table 3. Because of the low resolution of the images, the volume of pores and cracks incorporated in the yarns (micro-scale) was only estimated.

Tab. 3: Volume fraction of composite phases with basalt and carbon reinforcement

Level	*Phase*	*Volume fraction*	
		Basalt	*Carbon*
		[-]	[-]
Yarn	*Pores*	*0,01*	*0,01*
	Fibres	*0,665*	*0,688*
Composite	*Pores*	*0,103*	*0,119*
	Fibres	*0,444*	*0,47*

3.1.3. Material parameters measurement

The nanoindentation was utilized to obtain Young's moduli and Poisson's ratios for matrices and fibers of composites with basalt and carbon reinforcements. It has been performed in the Department of Mechanics, Faculty of Civil Engineering, CTU in Prague, by doc. Ing. Jiří Němeček, Ph.D.

It is a direct method for the measurement of micromechanical properties in a very small area and consists in pushing a diamond indentor perpendicularly into the surface of the measured sample at levels from nanometer to micrometer. Dependency of the applied force on the indentation depth is monitored during the test. The curves then allow for obtaining information about toughness, Young's elasticity modulus or viscoelastic properties of the measured sample. In our case the pyramidal indentor (Berkovich) was used. For detailed description of this method see (Němeček, 2010).

The results of measurements are summarized in Tables 4 and 5. Presented values are averages of each measurement. The values acquired from the literature are also provided, which allows us to verify the credibility of the acquired parameters.

Tab. 4: Basalt reinforced composite

Material parameter	*Nanoindentation*			*Literature*		
	Fibres		*Matrix*	*Fibres*		*Matrix*
	longitudinal	*diagonal*		*longitudinal*	*diagonal*	
E [GPa]	*45,8*	*18,09*	*2,12*	*80*	-	*2,3*
ν [-]	*0,4*	*0,24*	*0,24*	-	-	-

Tab. 5: Carbon reinforced composite

Material parameter	*Nanoindentation*			*Literature*		
	Fibres		*Matrix*	*Fibres*		*Matrix*
	longitudinal	*diagonal*		*longitudinal*	*diagonal*	
E [GPa]	*35,95*	*18,49*	*34,58*	*294*	*12,8*	*2,3*
ν [-]	*0,4*	*0,24*	*0,24*	*0,24*	*0,4*	-

It is evident from Tables 4 and 5 that some experimentally measured material properties differ from data found in the literature (Vorel, 2009; Černý). This is probably caused by the influence of the remaining phases, the treatment during the manufacturing process and the problematic experimental evaluation of fiber properties in the longitudinal direction. The input parameters used for the calculation of effective stiffnesses are listed in Table 6.

Tab. 6: Considered material parameters

Material parametres	*Basalt*		*Carbon*		*Matrix*
	Fibres		*Fibres*		
	longitudinal	*diagonal*	*longitudinal*	*diagonal*	
E [GPa]	*80*	*18*	*294*	*12,8*	*2,12*
G [GPa]	*25*	*10*	*11,8*	*4,6*	*0,85*
ν [-]	*0,24*	*0,4*	*0,4*	*0,24*	*0,24*

3.2. Evaluation of effective elastic properties by the Mori-Tanaka scheme

The Mori-Tanaka method belongs to the simplified analytical methods, which are based on micromechanics of continuum. Currently it is one of the most used mean field theories.

The method builds upon the solution of an isolated inclusion in an unbounded matrix subjected to a macroscopic stress or strain, in case of evaluation of the effective elastic properties. One of the assumptions of this method is mutual interaction of individual heterogeneities.

The Mori-Tanaka scheme is the explicit method and is known to provide quick and reliable estimates of the effective properties of composite systems when the information is limited to volume fractions and material properties of individual phases.

Now consider a n-phase composite - matrix with the subscript "*1*" or "*m*" and heterogeneities with subscript "*2,...n*". The subscript "*r*" is considered for any phase.

The relation between the local strain field of heterogeneities and the matrix can be written using the concentration factor $\mathbf{A}_r$ as

$$\boldsymbol{\varepsilon}_r = \mathbf{A}_r \, \boldsymbol{\varepsilon}_m \ . \tag{1}$$

Another concentration factor must be introduced to get the relation between $\boldsymbol{\varepsilon}_j$ and $\boldsymbol{E}$

$$\boldsymbol{\varepsilon}_r = \mathbf{A}_r^{MT} \boldsymbol{E} \ . \tag{2}$$

The relation between $\mathbf{A}_r$ and $\mathbf{A}_r^{MT}$ is provided by

$$\mathbf{A}_i^{MT} = \mathbf{A}_i \left(c_m \mathbf{I} + \sum_{r=2}^{N} c_r \mathbf{A}_r \right)^{-1} \quad . \tag{3}$$

The effective stiffness matrix attains the form

$$\mathbf{L}^{MT,ef} = \mathbf{L}_m + \sum_{r=2}^{N} c_r (\mathbf{L}_r - \mathbf{L}_m) \mathbf{A}_r^{MT} \quad . \tag{4}$$

Realizing its explicit format the application of the Mori-Tanaka method is extremely simple providing the concentration factors are available in closed form solution as is the case for many shapes of inclusions of ellipsoidal type. The method is particularly suitable for systems with well-defined matrix (Böhm, 2007).

The effective elastic properties were obtained by homogenization in four step homogenization process using the HELP software specially developed for these types of calculations. The results are summarized in Tables 7 and 8.

Tab. 7: Effective elastic properties of basalt reinforced composite

Material	*Equivalent inclusion*		*Effective elastic properties* [GPa]	
	Fibers/yarns $c_f = 0.66/0.44$	*Pores* $c_v = 0.01/0.10$	E_{11} , E_{22} , E_{33}	G_{12} , G_{23} , G_{13}
Fiber - matrix	*∞, 1, 1*	-	*54.3, 6.4, 6.4*	*2.7, 3.7, 3.7*
Porous yarn	→	*∞, 1, 1.6*	*53.8, 6.1, 6.2*	*2.6, 3.6, 3.6*
Yarns - matrix	*∞, 6.3, 1*	-	*15.5, 15.5, 4.0*	*1.4, 1.6, 1.6*
Porous composite	→	*3, 3, 1*	*12.7, 12.7, 3.2*	*1.2, 1.2, 1.2*

Tab. 8: Effective elastic properties of carbon reinforced composite

material	*Equivalent inclusion*		*Effective elastic properties* [GPa]	
	Fibers/yarns $c_f = 0.66/0.44$	*Pores* $c_v = 0.01/0.10$	E_{11} , E_{22} , E_{33}	G_{12} , G_{23} , G_{13}
Fiber - matrix	*∞, 1, 1*	-	*203.5, 5.9, 5.9*	*2.2, 3.4, 3.4*
Porous yarn	→	*∞, 1, 1.6*	*201.5, 5.7, 5.8*	*2.1, 3.3, 3.3*
Yarns - matrix	*∞, 6.3, 1*	-	*55.7, 55.7, 4.3*	*1.4, 1.6, 1.6*
Porous composite	→	*3, 3, 1*	*40.5, 40.5, 3.3*	*1.2, 1.3, 1.3*

3.3. Evaluation of effective elastic properties by finite element method

Nowadays FEM is considered to be the most universal method for solving variation formulated problems of physics connected to problems of field theory. One of the significant advantages of FEM in the field of continuum mechanics is particularly the possibility of solving tasks for universal geometric shape of body, universal load and support and also for complex constitutive relations of a material. Herein, FEM was adopted to solve the homogenization problem at the level of SEPUC.

Calculation has been performed in FELN software. This software enables a direct application of load in the form of macroscopically constant strains or stress. Unlike in commercial software here the primary unknown are fluctuation parts of the displacement field u^*, which enables simple input of periodical boundary conditions - identical displacements in opposite planes of statistically equivalent periodic unit cell - SEPUC.

Numerical determination of the effective elastic properties in case of one-step homogenization on meso-scale, which is in this paper represented by SEPUC, loaded by e.g. macroscopically homogeneous vector of strains, is performed according to the following formulas.

Local strain field can be decomposed into homogeneous, macroscopically linear part

$$U_i = E_{ij} x_j \tag{5}$$

and fluctuation part u^* such that

$$u_i(\boldsymbol{x}) = E_{ij} x_j + u_i^*(\boldsymbol{x}) \,. \tag{6}$$

Local strain thus becomes

$$\boldsymbol{\varepsilon}(\boldsymbol{x}) = \boldsymbol{E} + \boldsymbol{\varepsilon}^*(\boldsymbol{x}). \tag{7}$$

Introduction of local $\boldsymbol{\sigma}(\boldsymbol{x})$ and macroscopic Σ stress fields allows us to express the Hill Lemma as

$$\left\langle \boldsymbol{\varepsilon}(\boldsymbol{x})^{\mathsf{T}} \boldsymbol{\sigma}(\boldsymbol{x}) \right\rangle = \boldsymbol{E}^{\mathsf{T}} \boldsymbol{\Sigma} \,, \tag{8}$$

where $\langle f \rangle$ represents the volume average of a given value f .

Based on formula (8) it is possible to acquire the effective stiffness matrix of a given SEPUC from 6 independent elasticity solutions. For further details see (Vorel et al., 2013).

The calculation was preceded by the generation of finite element meshes. This was performed in ANSYS software. Effective stiffness acquired by FEM are summarized in Table 9.

Tab. 9: Effective elastic properties

Reinforcement material	*Effective elastic properties* [GPa]	
	E_{11} , E_{22} , E_{33}	G_{12} , G_{23} , G_{13}
Basalt	*12.8, 12.8, 0.3*	*1.7, 0.2, 0.2*
Carbon	*39.3, 39.3, 1.5*	*1.6, 0.7, 0.7*

3.4. Comparison of results

Tables 10 and 11 provide final effective elastic properties of the analyzed materials obtained by the Mori-Tanaka method, the finite element method and also by experimental measurements.

Tab. 10: Effective Young's elasticity moduli

Method	*Effective Young's elasticity moduli* [GPa]			
	Basalt reinforcement		*Carbon reinforcement*	
	E_{11} , E_{22}	E_{33}	E_{11} , E_{22}	E_{33}
Mori-Tanaka	*12.7*	*3.2*	*40.5*	*3.3*
FEM	*12.8*	*0.3*	*39.3*	*1.5*
EXP - warp	*16.3*	*3.8*	*42.2, 41.6*	*3.5*
EXP - weft	*7.9, 6.9*	*3.2*	*41.6, 47.7*	*3.5*

Tab. 11: Effective shear elasticity moduli

Method	*Effective shear elasticity moduli* [GPa]			
	Basalt reinforcement		*Carbon reinforcement*	
	G_{12}	G_{23} , G_{13}	G_{12}	G_{23} , G_{13}
Mori-Tanaka	*1.2*	*1.2*	*1.2*	*1.3*
FEM	*1.7*	*0.2*	*1.6*	*0.7*
EXP - warp	*11.5*	*1.7*	*35.9*	*0.7*
EXP - weft	*11.9*	*1.6, 1.8*	*32.5*	*1.2*

It is evident from Table 10 that the results delivered by all methods are comparable. In the transverse direction the results (Table 11) experimentally obtained seem to be influenced by large error when compared with the moduli in the longitudinal direction. This is especially evident from the shear modulus (G_{12}) for the composite with a carbon reinforcement. The error is probably caused by a low quality of fabricated samples and by the problematic experimental setup for this direction, see (Vorel et al., 2013).

4. Conclusion

Three particular approaches (two numerical and one experimental) were considered in this study to estimate the effective elastic properties of multi-layered textile composites. To suggest sufficient generality of the proposed multi-scale first-order homogenization based computational scheme we examined two different types of textile reinforcements.

Though the adopted experimental method can be used alone as a tool for the derivation of all nine components of the stiffness matrix, it is considered here merely for the validation of numerical predictions. These on the other hand may serve to check the reliability of the experimental results. Here, this comparison may suggest the inadequacy of the experimentally derived in-plane shear moduli G_{12}, particularly for carbon fabric reinforced samples, which are unrealistically high when compared to the in-plane tensile moduli E_{11} and E_{22}. The ultrasonic wave speeds propagating in the plane perpendicular to the plane (X_1;X_2) but rotated by the angle of 45° from the axis X_1 together with low thickness can make difficult to separate the quasi-longitudinal and the quasi-transverse modes. Therefore, the experimental identification of the G_{12} component is delicate. The quality of examined samples may also play an important role. These questions are opened and will be the subject of our future research. Nevertheless, the actual in-plane shear modulus can for sure be bounded by the experimental and numerical values.

Taking account of all the introduced simplifications in determining the effective elastic properties of composites investigated by the Mori-Tanaka scheme and the finite element method, the agreement with the values obtained by experimental measurements is relatively good, and therefore computer modeling in this case seems to be a very effective tool for saving time and money resources.

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