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PLASTIC RESISTANCE OF ALUMINUM I-PROFILE UNDER BENDING AND TORSION ACCORDING TO CONTINUOUS STRENGTH METHOD

I. Baláž[*], Y. Koleková[**]

Abstract: *The continuous strength method (CSM) is a deformation-based design approach that allows a rational exploitation of strain hardening. This paper describes the development of the method and its application to aluminium structural elements. The key design concepts are expressed through a set of straightforward design equations, while the range of stress-strain responses is allowed for through material specific coefficients in the adopted bi-linear (elastic, linear hardening) material model. The design method enables enhancements in structural efficiency and, unlike traditional approaches, provides the designer with information on the level of plastic deformation that the structure is undergoing at the ultimate limit state. Plastic bending moment resistance utilizing strengthening in stress-strain diagram. Investigation of 6 stress-strain diagrams. Resistance of I-profile under combination of major axis bending moment and bimoment.*

Keywords: Aluminium, Deformation-based, Local buckling, Stainless steel, Steel, Strain hardening, Structures, σ-ε diagrams. I-profile resistances, Bending moment, Bimoment interaction.

1. Introduction

The cross-section and member design rules given in Eurocode EN 1993 and Eurocode EN 1999 are based on the assumption of elastic, perfectly plastic material behaviour leading to the concept of cross-section classification, the use of elastic and plastic moment capacities and plastic hinge design. In reality, structural steel and other structural metallic materials such as stainless steel and aluminium do not exhibit this form of idealised stress-strain response. Instead, the stress-strain curves of these materials display differing degrees of nonlinearity, roundedness in the region of the yield stress, a range of plateau lengths and often the absence of a plateau altogether, varying strain hardening slopes and so on. The idealisation of elastic, perfectly plastic material behaviour is generally reasonable for hot-finished structural steel with a long yield plateau, while for other materials, the idealisation is more questionable.

For the case of the traditional elastic, perfectly plastic material model, post-yield strains do not result in any increase in stress. However, for a hardening material model, increasing post yield strains do lead to an increase in stress, and hence the strength of a cross-section is related to the level of strain it can endure prior to failure, typically by inelastic local buckling. In such circumstances (i.e. design of structures composed of strain hardening materials), since strength is dependent on deformation, a deformation-based design approach becomes desirable. Recent research into such an approach, referred to as the continuous strength method (Gardner, 2008), is outlined in (Gardner, 2016).

The continuous strength method (CSM) is a deformation-based design approach that accounts for strain hardening. The method has been shown to give a high level of accuracy and consistency in predicting the resistance of structural steel (Gardner, 2008, Liew and Gardner, 2015), stainless steel (Afshan and Gardner, 2013, Zhao et al, 2015), and aluminium (Su et al, 2014) cross-sections under compression, bending and combined loading. The method has also been applied to the determination of cross-section resistances in fire (Theofanous et al, 2016).

The CSM has two key components: (i) a 'base curve' that defines the limiting strain ε_{CSM} for a cross-section (i.e. the deformation capacity) based on its local slenderness and (ii) a strain hardening material

[*] Prof. Ing. Ivan Baláž, PhD.: Department of Metal and Timber Structures, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Slovak Republic, Radlinského 11; 810 05, Bratislava; SK, ivan.balaz@stuba.sk

[**] Assoc. Prof. Ing. Yvona Koleková, PhD.: Department of Structural Mechanics, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Slovak Republic, Radlinského 11; 810 05, Bratislava; SK, yvona.koelkova@stuba.sk

model, which enables stresses greater than the yield stress f_y to be achieved. These two key components are described in (Gardner, 2016) and applied in this paper.

Since the continuous strength method is a deformation-based design approach, it requires the determination of a relationship between the maximum limiting strain that a cross-section can endure prior to reaching its ultimate capacity and its local slenderness. This relationship is equivalent to the process of cross-section classification used in many structural metallic design codes, but instead of placing a cross-section into a discrete behavioural class, a normalized limiting strain is assigned (Gardner, 2016). For slender cross-sections, the limiting strain is below the yield strain and there is therefore no benefit to be gained from strain hardening, except in the case of non-doubly-symmetric cross-sections that can have limiting strains on the compression side less than the yield strain yet take benefit from strain hardening on the tensile side; for non-slender sections, the limiting strain is beyond the yield strain and benefit can be derived from strain hardening. Cross-sections comprising flat plates and circular hollow sections (CHS) are described in (Gardner, 2016).

2. Stress-strain relationships of aluminium alloys

The models for the idealization of the stress-strain relationship of aluminium alloys are provided by the Annex E (EN 1999-1-1, 2007). These models are conceived in order to account for the actual elastic-hardening behaviour of such materials. The analytical characterization of the stress σ – strain ε relationship of an aluminium alloy can be done by means of one of the following models: (i) pieceswises models, (ii) continuous models.

All models defined in (EN 1999-1-1, 2007) were analyzed and compared with CSM model (Gardner, 2016). In this investigation the officially published Amendments A1 (2009) and A2 (2013) were taken into account together with proposal for Amendment (N 503, 2017). The results of the investigation for the wrought aluminium alloy EN AW-5083-O/H111 are given in Fig. 1. The aluminium alloy 5083 is known for exceptional performance in extreme environments. It is highly resistant to attack by both seawater and industrial chemical environments. The aluminium alloy 5083 also retains exceptional strength after welding. It has the highest strength of the non-heat treatable alloys but it is not recommended for use in temperatures in excess of 65 °C. It is typically used in: shipbuilding, rail cars, vehicle bodies, tip truck bodies, mine skips and cages, pressure vessels.

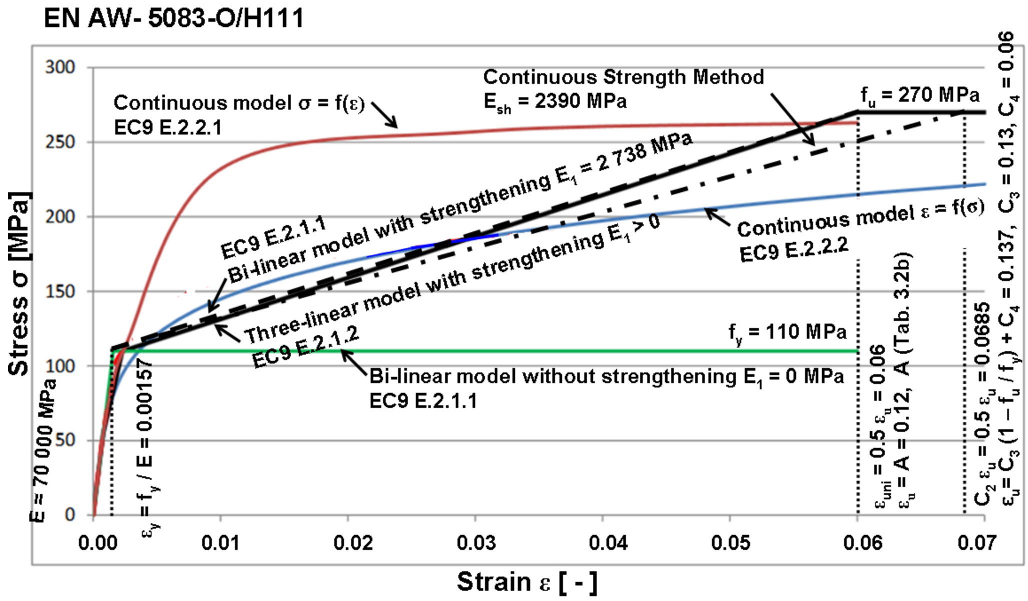

Fig. 1: Comparisons of the five stress-strain relationships given in EN 1999-1-1 with CSM one defined in (Gardner, 2016) for the non-heat treatable wrought aluminium alloy EN AW-5083-O/H111.

From the obtained results (Fig .1) it is clear that: (i) the continuous model $\sigma = f(\varepsilon)$ defined in the clause E.2.2.1 have to be corrected (Baláž 2017b,c), (ii) end part of the CSM diagram defining ε_u should be verified by more experiments (Baláž, 2017d, Mei-Ni et al, 2016), (iii) there is acceptable agreement between CSM model (dot-and-dashed line) and bi-linear model with strengthening defined in the clause E.2.1.1 (dashed line).

3. Continuous Strength Method (Gardner, 2016)

Within the continuous strength method (CSM), the base curve defines the relationship between cross-section deformation capacity and cross-section slenderness. For non-slender cross-sections, the base curve for cross-sections comprising flat plated elements (e.g. I-sections and square and rectangular hollow sections – SHS and RHS) is given by Equation 1 and shown with corresponding test data in Fig. 1. The test data have been collected from the literature and include results for steel, stainless steel and aluminium stub columns and beams (in-plane four-point bending). The transition between non-slender and slender cross-sections is defined at a local slenderness value of 0.68. Cross-sections with a local slenderness below 0.68 are deemed to be non-slender and have failure strains greater than the yield strain and can thus benefit from strain hardening; cross-sections with a local slenderness greater than 0.68 are slender and have limiting strains below the yield strain.

4. Application of Continuous Strength Method (CSM)

The method (Vlasov, 1936, Streľbickaja, 1947, 1958) together with CSM (Gardner, 2016) was applied by (Baláž, 2017a) for calculation of the resistance of I-section and channel section under combination of major axis bending moment M_y and bimoment B using: (i) elastic theory, (ii) plastic theory without strengthening (see model E.2.1.1 in Fig. 1), (iii) plastic theory with strengthening (see CSM model in Fig. 1).

The following numerical example was calculated for input values: extruded I-profile with dimensions h = 200 mm, b_f = 100 mm, t_f = 11.4 mm, t_w = 7 mm, made of wrought aluminium alloy EN AW-5083-O. The below values were calculated according to (Gardner, 2016). The local slenderness of the web and of the flange show that I-section is non-slender:

$$\bar{\lambda}_{p.w} = 0.216,\ \bar{\lambda}_{p.f} = 0.259,\ \bar{\lambda}_p = \max(0.216, 0.259) = 0.259 < 0.68 \qquad (1)$$

$$f_y = 110 \text{ MPa},\ f_u = 270 \text{ MPa},\ \varepsilon_y = f_y / E = 110 \text{ MPa} / 70\,000 \text{ MPa} = 0.00157 \qquad (2)$$

$$\varepsilon_u = C_3\,(1 - f_y / f_u) + C_4 = 0.13\,(1 - 110 \text{ MPa} / 270 \text{ MPa}) + 0.06 = 0.137 \qquad (3)$$

$$E_1 = (f_u - f_y) / (C_2\,\varepsilon_u - \varepsilon_y) = (270 \text{ MPa} - 110 \text{ MPa}) / (0.5 \text{ x } 0.137 - 0.00157) = 2\,390 \text{ MPa} \qquad (4)$$

$$\frac{0.25}{\bar{\lambda}_p^{3.6}} = 32.2,\ C_1 \frac{\varepsilon_u}{\varepsilon_y} = 0.5\ \frac{0.137}{0.00157} = 43.6,\ \varepsilon_{CSM} = \min(15, 32.2, 43.6)\,\varepsilon_y = 15 \text{ x } 0.00157 = 0.02357 \qquad (5)$$

CSM bi-linear diagram in Fig. 1 is defined by 3 points with coordinates as follows: the bottom point (0, 0), the middle point (ε_y, f_y), the top point ($C_2\ \varepsilon_u$ = 0.5 x 0.137 = 0.06852, f_u). There is acceptable difference comparing with EN 1999-1-1 bi-linear diagram according to clause E.2.1.1, which differs only in a coordinate of top point (ε_{uni} = 0.5 A = 0.5 x 0.12 = 0.6, f_u). Limiting values according to CSM and EN 1999-1-1, respectively are:

$$\varepsilon_{CSM.max} = \min(15\,\varepsilon_y,\ C_1\,\varepsilon_u) = \min(0.02357, 0.06852) = 0.02357 \qquad (6)$$

$$\varepsilon_{uni.max} = 0.30 - 022\,f_y / 400 \text{ MPa} = 0.2395 \text{ for } f_y < 400 \text{ MPa, and } 0.08 \text{ for } f_y \geq 400 \text{ MPa} \qquad (7)$$

The characteristic value of the plastic bending moment resistance calculated for bi-linear model without strengthening (Fig. 1) is

$$M_{y.pl.Rk} = W_{y.pl}\,f_y = 277.251 \text{ cm}^3\ 110 \text{ MPa} = 30.5 \text{ kNm} \qquad (8)$$

The characteristic value of the plastic bending moment resistance calculated for bi-linear model with strengthening (Fig. 1):

a) according to formula (F.2) and Table F.2 in EN 1999-1-1 is M_{EC9} = 31.44 kNm,
b) according to formula (37.9) in (Streľbickaja, 1958) is M_{Str} = 39.10 kNm,
c) according to CSM formula (13) in (Gardner, 2016) is M_{CSM} = 43.97 kNm.

5. Conclusions

Application of CSM in resistance calculation of I-profile under interaction of bending moment and torsion. Comparisons of five strain-strain diagrams given in EN 1999-1-1 with CSM one (Gardner, 2016).

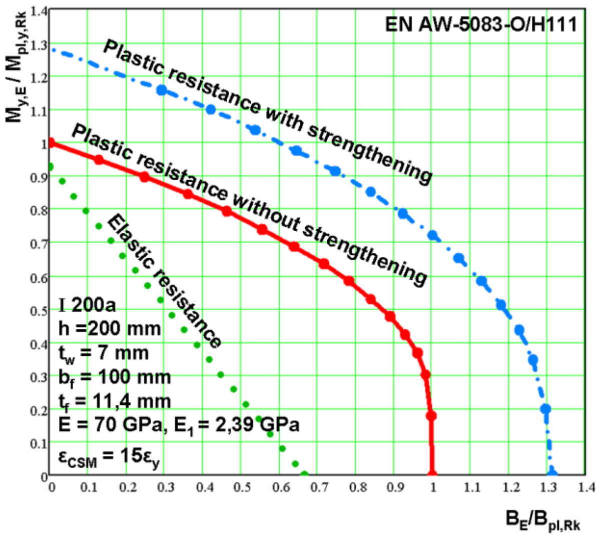

Fig. 2: Resistance of I-section under interaction of bending moment and bimoment.

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