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COMPUTATIONAL PERFORMANCE OF A DSG-BASED ISOGEOMETRIC BEAM ELEMENT

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Abstract: *An application of Discrete shear gap (DSG) method to the isogeometric Timoshenko beam element with variable curvature is presented. A locking-removal capability of DSG is compared to the reduced integration. While the reduced integration does not remove stress or strain oscillations, DSG provides results matching exact solution. The application of DSG method results in full stiffness matrix of a patch and compared to reduced integration is computationally more demanding. This possibly leads to a deterioration of overall time efficiency of the isogeometric approach. Thus the computational performance of the element is compared to the standard straight beam element. Results proved the enormous time efficiency of isogeometric element over standard FEA and excellent convergence properties of DSG method.*

Keywords: Beam element, Discrete shear gap, Isogeometric analysis, NURBS, Shear locking.

1. Introduction

Isogeometric analysis (Hughes et al., 2005) is a recently developed alternative of standard finite element method, which has been proposed to bridge the gap between CAD (Computer Aided Design) and FEA (Finite Element Analysis). In practice, CAD models are mostly represented by the splines, while the standard FEA is usually based on polynomial basis functions. Isogeometric analysis uses the spline basis for both CAD and FEA data representations.

The switch of basis functions from polynomials to splines in FEA offers great benefits. The same geometry representation can be shared by both CAD and FEA systems and thus no transformation from one to another is needed. Moreover spline functions enable exact description of the shapes which cannot be exactly described by polynomials (e.g. conic sections). These aspects can significantly improve computational efficiency and accuracy.

An exact geometry representation in isogeometric analysis can be especially profitable for structures of curved geometries. Main focus of this paper is placed on structural analysis of curved beams. Timoshenko beam element based on work of Bouclier (2012) is presented. The formulation of the element suffers from locking phenomena and DSG method is used to unlock the element. Finally, the computational efficiency over classical straight beam element is studied.

2. Isogometric beam element

The presented element formulation uses NURBS (Non-Uniform Rational B-Splines) as a basis functions for both geometry description and unknown approximations. A p^{th} degree NURBS functions N_i^p are generated from B-splines as

$$N_i^p(\xi) = \frac{S_i^p(\xi) w_i}{\sum_{\hat{i}=1}^{n} S_{\hat{i}}^p(\xi) w_{\hat{i}}}, \tag{1}$$

where S_i^p are p^{th} degree B-spline functions, w_i are the weights associated with the corresponding basis function and $\xi \in (0, 1)$ is a parametric coordinate running through the entire patch (subdomain of knot

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spans which are seen as “elements” in isogeometric analysis). See Piegl (1997) for better understanding of NURBS geometry.

A curved Timoshenko beam element with three independent unknowns, tangential displacement $u_t(s)$, normal displacement $u_n(s)$ and rotation $\theta(s)$, is being considered. Membrane, transverse shear and bending strains are given by

$$\varepsilon_m(s) = u_t'(s) - \frac{u_n(s)}{R(s)}, \quad \gamma_s(s) = \frac{u_t(s)}{R(s)} + u_n'(s) - \theta(s), \quad \chi_b(s) = \theta'(s), \qquad (2)$$

where curvilinear coordinate s runs along the midline of the beam, the prime indicates a derivation with respect to the s and R is a radius of a curvature. For simplicity, the dependence on (s) will be omitted in the following text. The stiffness matrix is evaluated using

$$\boldsymbol{K} = \int_0^L \boldsymbol{B}^T \boldsymbol{D} \boldsymbol{B}\, ds, \qquad (3)$$

where strain-displacement matrix $\boldsymbol{B}$ is derived using formulas for strain components (2) and $\boldsymbol{D}$ is a material matrix resulting from

$$N = EA\varepsilon_m, \qquad Q = GA\gamma_s, \qquad M = EI\chi_b, \qquad (4)$$

where N, Q and M are axial force, transverse shear force and bending moment, respectively. Young’s modulus E, shear modulus G, area A and moment of inertia I are the material and cross-section characteristics.

3. Numerical locking

Due to the independent approximation of displacements and rotation the element suffers from shear locking. The formula for bending strain results in lower order term than formula for shear strain (2), but actually this should be vice-versa. Moreover from the formula for the shear strain it is obvious that zero shear strain cannot be satisfied along entire patch when the same order interpolation of unknowns is used because of field-inconsistency (Adam et al., 2014). In this paper the performance of Discrete Shear Gap (DSG) method, originally developed for unlocking the standard finite elements (Bletzinger, 2000) and further extended also for isogeometric elements (Echter, 2010), is examined.

The DSG approach can be divided into several steps yielding the modified strain-displacement matrix $\boldsymbol{B}$ used to evaluate stiffness matrix $\boldsymbol{K}$. The main idea of the method is to satisfy the equation for shear strain (2) in integral sense (instead of pointwise). The shear contributions $u_n^{\gamma^{h_i}}$ (so called “shear gaps”) to the deflection u^n in the collocation points are calculated by integration of γ_s^h (2) as

$$u_n^{\gamma^{h_i}} = \int_0^{s_i} \gamma_s^h\, ds = \int_0^{s_i} \frac{u_t}{R} + u_n' - \theta\, ds = \boldsymbol{B^{DSG}}\boldsymbol{r}, \qquad (5)$$

where the collocation points s_i are given as Greville abscissa of the control points (Piegl, 1997). The modified shear displacements $u_n^{\gamma^{mod^h}}$ are interpolated using NURBS basis functions

$$u_n^{\gamma^{mod^h}} = \sum_{i=1}^{n} N_i \tilde{u}_n^{\gamma^{h_i}}. \qquad (6)$$

In case of isogeometric analysis, the discrete shear gaps $\tilde{u}_n^{\gamma^h}$ are non-interpolatory, therefore they need to be expressed using values in the control points. For this purpose, the transformation matrix $\boldsymbol{A}$ is derived

$$\left\{u_n^{\gamma^h}\right\} = \boldsymbol{A} \left\{\tilde{u}_n^{\gamma^h}\right\}, \quad A_{ij} = N_j(s_i), \qquad (7)$$

where $\left\{u_n^{\gamma^h}\right\}$ are interpolatory values of shear gaps at control points and $N_j(s_i)$ is the j^{th}-basis function evaluated at i^{th}-collocation point. The modified shear strain is then given as

$$\gamma_s^{mod^h} = \sum_{i=1}^{n} N_i' \tilde{u}_n^{\gamma^{h_i}} \tag{8}$$

and the modified part of the strain-displacement matrix $\boldsymbol{B}$ corresponding to shear component is obtained by combining (5)-(8) as

$$\boldsymbol{B}^{\gamma} = \boldsymbol{N}'\boldsymbol{A}^{-1}\boldsymbol{B}^{DSG}. \tag{9}$$

It is important to note, that this modification has to be performed on the patch level, as the collocation points are located along the entire patch. Moreover the inverse of $\boldsymbol{A}$ introduce a full global patch stiffness matrix. This leads to the higher computational cost which could possibly reduce the advantages of isogeometric approach and therefore should be further analysed.

4. Numerical examples

The presented isogeometric beam element has been implemented into OOFEM finite element code (Patzák, 2017) and its performance has been tested on the circular cantilever beam subjected to the tip force load (Fig. 1). Ability of reduced integration and DSG method to unlock the element with cubic approximation is illustrated in Fig. 1. For reduced integration, the scheme proposed by Bouclier (2012) has been used (i.e. two Gauss points per each knotspan + 2 additional Gauss points per patch). Both methods (DSG and reduced integration) show good results when the convergence of normal displacement at the tip of the beam is studied, nevertheless the reduced integration still suffers from the oscillations in strains along the beam (Fig. 1). The DSG method proven itself to successfully unlock the element and provide results in agreement with exact solution.

In order to document the benefits of IGA over standard FEA, the computational performance of isogeometric element using DSG approach has been compared to classical straight Timoshenko beam element with cubic approximation which does not suffer from locking (Bittnar, 1992). To demonstrate the quality of solution, the L^2 norm of normal displacement error $\|e\|$

$$\|e\| = \sqrt{\int_0^L \left(u_n^{exact} - u_n^h\right)^2 \, ds}, \qquad u_n^{exact} = \left(\frac{R^3}{2EI} + \frac{R}{kGA} + \frac{R}{2EA}\right)\varphi \, sin(\varphi) \tag{10}$$

is used. To illustrate the convergence, the solution time consumed with respect to the error $||e||$ is plotted in Fig. 2. The obtained results document the enormous time efficiency of isogeometric element over standard FEA. Moreover, it is obvious, that degree elevation can reduce the error while the computational time is kept low.

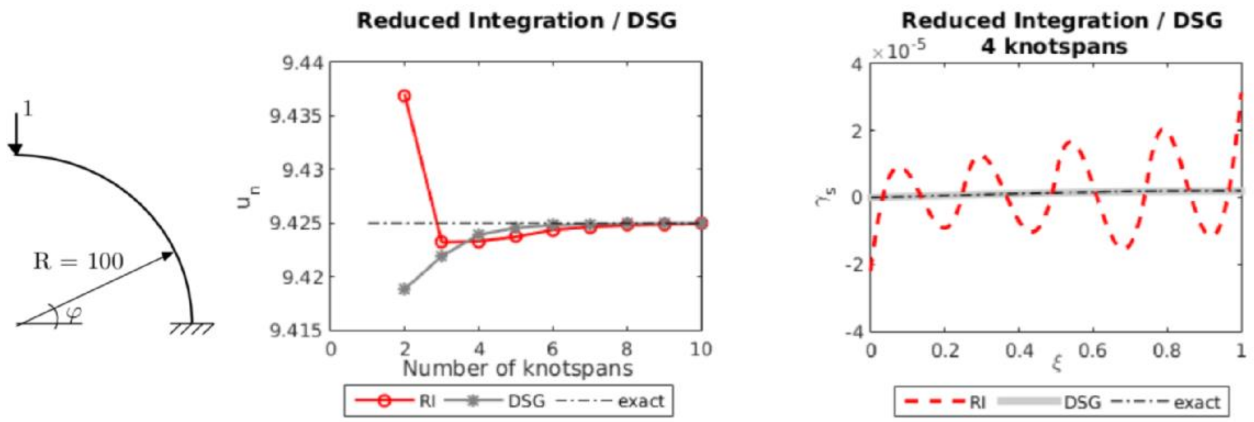

Fig. 1: Circular cantilever beam: (left) problem setup, $E = 10^6, h = 1.0, b = 1.0, \nu = 0.0$, (centre) convergence of the normal displacement at the tip of the beam with cubic NURBS approximation using reduced integration (2 Gauss points per knotspan + 2 additional Gauss points per patch), (right) oscillations in shear strain when the reduced integration is used while the use of DSG matches the exact solution.

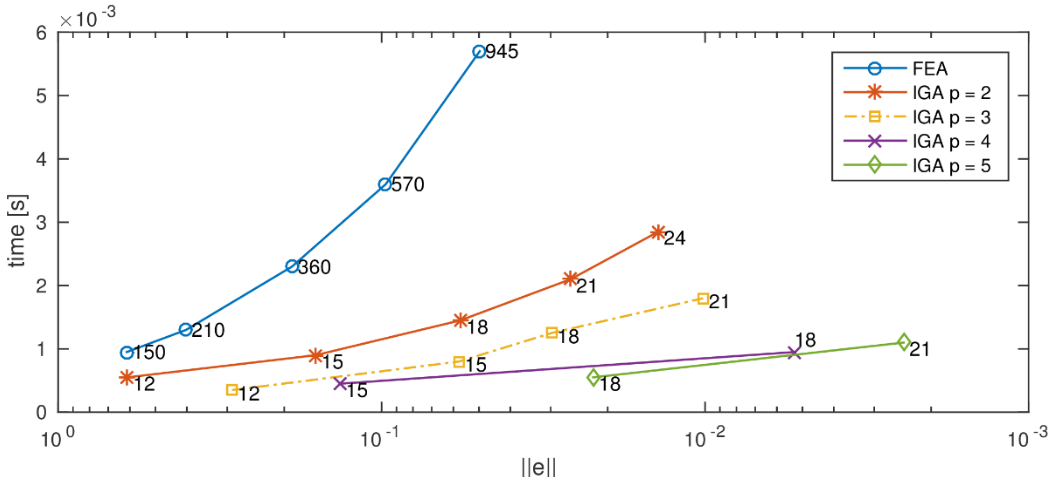

Fig. 2:Circular cantilever beam: Comparison of solution time in OOFEM with respect to the error of normal displacement using Timoshenko beam elements with cubic approximation (FEA) and isogeometric beam elements with different approximation orders. The numbers in the plot indicate the required number of degrees of freedom for specific simulation.

5. Conclusions

The isogeometric Timoshenko beam element has been implemented into existing finite element code. It has been shown, that the reduced integration does not remove the oscillations in strains, while the satisfactory results are obtained in case of use of DSG method.

The significant time efficiency of isogeometric element over standard straight beam element has been proven. The performance of standard FEA could be enhanced using suitable curved beam element formulation, however such is not currently available in OOFEM. Also these formulations have usually some assumptions, such as constant curvature. On the contrary, isogeometric formulation enable exact description of arbitrarily curved geometries.

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