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INSTABILITTY OF A STEPPED COLUMN LOADED BY THE EXTERNAL FOLLOWER FORCE

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Abstract: *In this paper the instability problem of a stepped column loaded by the follower force is discussed. The column is composed of three sections with different cross-sectional areas. The connection of the each section is realized by means of the pins strengthened by the rotational springs. This type of connection allows one to simulate the different method of assembly of sections or even the defect (attenuation) in the connection. The boundary problem of the instability of the presented slender system has been formulated on the basis of the Hamilton's principle and solved with small parameter method. The results of the numerical simulations are concern on the shape of the characteristic curves under an influence of the different cross sectional areas of the segments.*

Keywords: Column, Flutter, Instability, Boundary Problem, Follower Force.

1. Introduction

In this paper the instability of the slender system in the form of the column loaded by the follower force is presented. The introduced load causes that the column loses stability via divergence or flutter, depending on the follower factor. According to the nomenclature proposed by Kordas and Życzkowski (1963) the discussed type of external load can be classified as: $\eta < 0$ – anti-follower force, $0 < \eta < 1$ – sub-follower force, $\eta = 1$ – follower force, $\eta > 1$ – super-follower force.

The studies on slender structures loaded by the follower force have been done by Przybylski and Sokół (2012). Authors have investigated the multimember column integrated with piezoceramic element. The piezo-rod induced the additional compressive or tensional forces, what lead to pre-stressing of the column and finally the shift of divergence/flutter instability regions has been obtained. Kukla (2009) have investigated the stepped cracked column. The size of the crack has been calculated on the basis of presentation written by Ostachowicz and Krawczuk (1991). Kukla has proposed the solution of the boundary problem by means of Green's functions and obtained the characteristic curves of the cracked structure. Zamorska et.al (2015) have presented the analytical simulation done with Green's function of the cracked beam with variable cross-sectional area and the numerical ones from FEM module of CATIA software. In the same time Cekus and Waryś (2015) have studied an identification of parameters of discrete-continuous models in the form of stepped columns. The chosen parameters of the systems have been found on the basis of free vibration frequency magnitude obtained from experimental studies. The proposed mathematical model has been solved according to the Lagrange multiplier formalism, while an optimization based on the genetic algorithm. The studies on the divergence/flutter instability regions have been also done by Tomski and Uzny (2011). Authors have studied a column with constant cross sectional area strengthened by the spring at the fixed end and loaded by the generalized Beck's load and obtained the regions of divergence/flutter instability. Continuation of the studies has been presented in 2013.

In this paper the stepped column loaded by follower force is investigated. The proposed structure is composed of three sections connected by the pins strengthened by the rotational springs. This type of connection allows one to simulate the different methods of assembly of sections. The presented results are concern on the shape of characteristic curves (external load – vibration frequency) and divergence/flutter instability regions in relation to the different cross-sectional areas of sections.

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2. Boundary problem formulation

The investigated system is presented in the Fig. 1. As shown, the structure is loaded by external force P localized on the top end of the cantilever column. Sections have lengths l_1, l_2, l_3. Even and odd sections are connected by rotational springs.

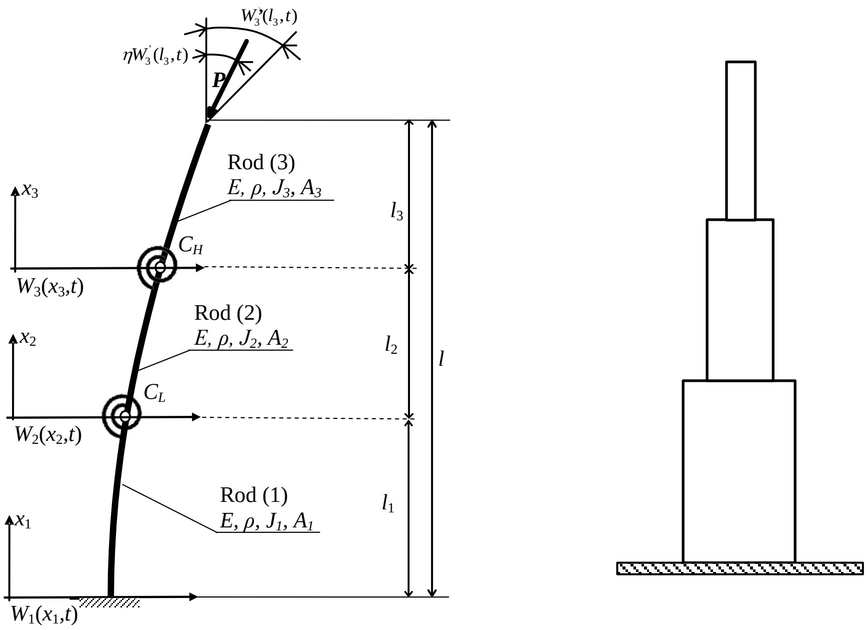

Fig. 1: Investigated system.

The boundary problem has been formulated on the basis of the Hamilton' principle:

$$\delta\int_{t_1}^{t_2}(T - V + L_c)dt + \int_{t_1}^{t_2}\delta L_n dt = 0 \tag{1}$$

The kinetic and potential energies and components of work of conservative and non-conservative forces are as follows:

$$V = \frac{1}{2}\left\{\sum_{i=1}^{3}\int_0^{l_i} EJ_i\left[\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}\right]^2 dx + \int_0^{l_i} EA_i\left[\frac{\partial U_i(x_i,t)}{\partial x_i} + \frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]^2 dx\right\} +$$

$$+\frac{1}{2}C_H\left(\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|_{x_3=0} - \left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=l_2}\right)^2 + \frac{1}{2}C_L\left(\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=0} - \left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=l_1}\right)^2$$

$$T = \frac{1}{2}\sum_{i=1}^{3}\int_0^{l_i}\rho A_i\left(\frac{\partial W_i(x_i,t)}{\partial t}\right)^2 dx \quad L_c = -PU_3(l_3,t)\,,\; L_n = \eta P\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|_{x_3=l_3} W_3(l_3,t) \tag{2a, d}$$

On the basis of the Hamilton's principle and integration and variation operations inter alia the equations of motion in transversal direction are found:

$$EJ_i W_i^{IV}(x_i,t) + PW_i^{II}(x_i,t) + \rho A_i \ddot{W}_i(x_i,t) = 0 \quad i = 1,2,3 \tag{3}$$

The natural and geometrical boundary conditions are as follows:

$$W_1(0,t) = W_1^I(0,t) = EJ_3 W_3^{II}(l_3,t) = 0\,, \quad W_j(l_j,t) = W_{j+1}(0) \;\; j = 1,2$$

$$EJ_j W_j^{III}(l_j,t) + S_j W_j^I(l_j,t) - EJ_{j+1}W_{j+1}^{III}(l_{j+1},t) - S_{j+1}W_{j+1}^I(l_{j+1},t) = 0 \quad j = 1,2$$

$$-EJ_j W_j^{II}(0,t) + C_{L/H}(W_j^I(0,t) + W_{j-1}^I(l_{j-1},t)) \;\; j = 2,3 \;\; EJ_j W_j^{II}(l_j,t) - C_{L/H}(W_{j+1}^I(0,t) + W_j^I(l_j,t)) \;\; j = 1,2$$

$$EJ_3 W_3^{III}(l_3,t) + P(1-\eta)W_3^I(l_3,t) \tag{4a – l}$$

The presented problem can be solved numerically by introduction of Eq. (3) into boundary conditions (4a – l). The obtained matrix determinant equated to zero leads to the transcendental equation on the basis of which the external load – natural vibration frequency relationship is computed.

3. Results of numerical simulations

The discussion of the results of numerical simulations is done by means of the non-dimensional parameters:

$$p = \frac{Pl^2}{EJ_1}, c_H = \frac{C_H l}{EJ_1}, c_L = \frac{C_L l}{EJ_1}, d_i = \frac{l_i}{l}, \mu_{21} = \frac{EJ_2}{EJ_1}, \mu_{32} = \frac{EJ_3}{EJ_2}, \omega = \sqrt{\Omega^2 \frac{\rho A_1 l^4}{EJ_1}}, \qquad (5a – f)$$

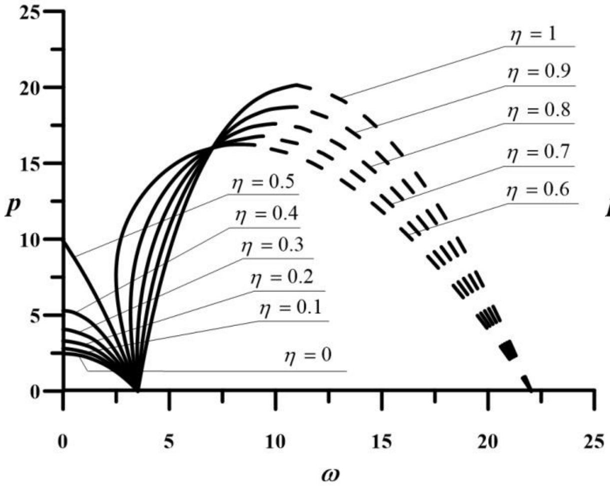

Fig. 2: Influence of cross sectional area on the shape of characteristic curves ($c_H = c_L = 10^3$, $d_1 = d_2 = d_3 = 0.33$, $\mu_{21} = \mu_{32} = 1$).

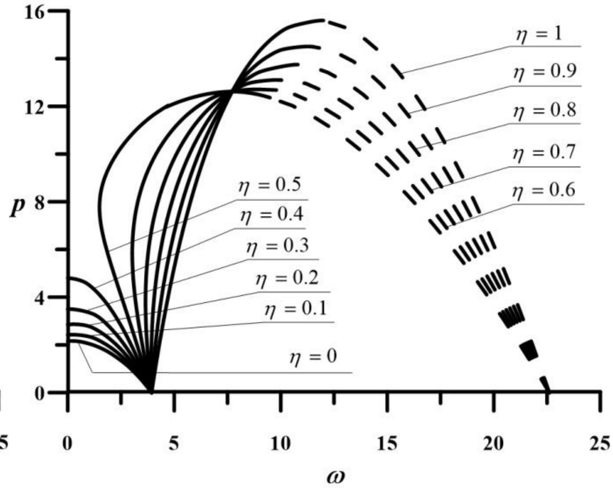

Fig. 3: Influence of cross sectional area on the shape of characteristic curves ($c_H = c_L = 10^3$, $d_1 = d_2 = d_3 = 0.33$, $\mu_{21} = 0.75$, $\mu_{32} = 1$).

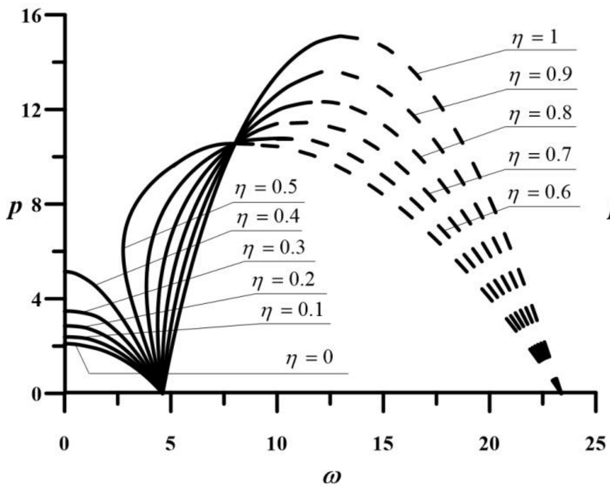

Fig. 4: Influence of cross sectional area on the shape of characteristic curves ($c_H = c_L = 10^3$, $d_1 = d_2 = d_3 = 0.33$, $\mu_{21} = 0.75$, $\mu_{32} = 0.5$).

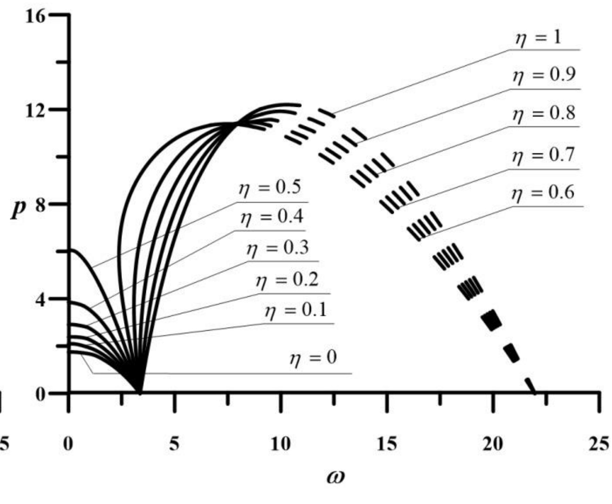

Fig. 5: Influence of cross sectional area on the shape of characteristic curves ($c_H = c_L = 10^3$, $d_1 = d_2 = d_3 = 0.33$, $\mu_{21} = 0.5$, $\mu_{32} = 2$).

The presentation of the results has been started from the case when all the segments have equal cross-sectional area (Fig. 2). That allows on to check the validity of the proposed mathematical model. The column loses stability by divergence up to $\eta = 0.5$. Above that magnitude the flutter instability occurs. The highest flutter external load magnitude is 20.05; divergence 9.86. The reduction of cross sectional area of segments 2 and 3 (Fig. 3) results in an increase of first as well as second vibration frequency magnitude. At this configuration the reduction of divergence instability region has been noticed – the column changes divergence instability to flutter one at $\eta > 0.4$ in relation to the Fig. 1 configuration. The change of cross-sectional area causes also the reduction of divergence and flutter maximum loading. The location of common point (point in which the characteristic curves at flutter instability are crossing) is shifted to the higher vibration frequency and is present at lower external load magnitude. The further reduction of the cross-sectional area of the top segment (Fig. 4) results in further maximum loading capacity drop, increase of free vibration frequency and shift of the common point (present at higher vibration frequency and at lower external load). The change of instability type occurs at $\eta > 0.4$. In the last case (Fig. 5) when the first and top segments have equal cross-sectional areas and the middle one is thinner than the others the lowest loading capacity has been observed. The free vibration frequency has lowered in relation to the Figs. 3 and 4. The instability change has now shifted to $\eta > 0.5$. At flutter instability the difference in maximum loading is the smallest from the all presented cases.

4. Conclusions

The following conclusions have been made on the basis of the results of the numerical simulations:

- the control of divergence/flutter instability regions can be done by means of follower factor or by change of cross-sectional area of chosen segments,
- the maximum loading as well as vibration frequency can be controlled both by the follower factor and cross-sectional area,
- the location of common point at flutter instability changers in relation to the cross-sectional area of segments,

In the future studies the different length of the elements as well as the connection stiffness between sections should be considered. Furthermore the higher components of natural vibration frequency can be solved by means of small parameter method. The verification of the proposed mathematical model can be done during the experimental studies, where the follower load is realized by the small scale jet engine.

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