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ALL-PASS FILTERS AS A TOOL FOR CONVERTING A POSITIVE FEEDBACK TO A NEGATIVE FEEDBACK WHEN CONTROLLING WEAKLY DAMPED SYSTEMS

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Abstract: *The problem of active vibration control of weakly damped mechanical structures is potentially unstable modes of vibrations due to the positive feedback for some vibration modes. The paper will discuss the change of positive feedback on the negative one using all-pass discrete-time filters which are arranged in a cascade. The piezoelectric actuator as a source of force is used to damp vibration. It is well known that this actuator type has hysteresis.*

Keywords: Active vibration control, All-pass filter, Cantilever beam.

1. Introduction

The problem of the active vibration control of weakly damped mechanical structures consists in potentially unstable modes of vibrations. If the gain feedback is increased, then some of the poles of the transfer function recedes the stability boundary which is the imaginary axis, while the other poles approach it, even the boundary is crossed for a large feedback gain, and the system becomes unstable due to the positive feedback for these vibration modes. The paper will discuss the change of positive feedback to the negative one using all-pass discrete-time filters which are arranged in a cascade. Theory will be illustrated by an example of the active vibration control of the cantilever beam.

2. Model of a mechanical structure

As assumed the paper deal with the mechanical systems of *N* degrees of freedom. The properties of these mechanical systems describe the equation of motion. In addition to this equation, the system can be described by a modal or experimental model. The modal model represents the modal matrix **U**, which is a matrix of eigenvectors, and spectral matrix **Λ**, which is a diagonal matrix with the eigenvalues on the main diagonal. The experimental model is represented by measured frequency response functions (matrix **H**). The frequency transfer functions may be formed from all types of models as follows

$$H_{r,q}(\omega) = \sum_{n=1}^{N} \frac{v_{n,r} v_{n,q}}{\Omega_n^2 - \omega^2 + j2\xi_n \Omega_n \omega}, \quad r,q = 1,2,\ldots,N \tag{1}$$

where $v_{n,r}, v_{n,q}, n,r,q = 1,\ldots,N$ are the elements of the *N*-dimensional normalized eigenvector $\mathbf{v}_n = [v_{n,1},\ldots,v_{n,N}]^T$ which is associated with the natural frequency Ω_n and relative damping ξ_n. The coordinates $v_{n,1}, v_{n,2},\ldots,v_{n,N}$ determine a vibration or mode shape. The transfer function (1) is a sum of the transfer functions of the second order systems which correspond to the mode indexed by *n*. The product $k_n = v_{n,r} v_{n,q}$ for given value of indexes *r* and *q* is called a modal constant and depends on the modal shapes for the natural frequency Ω_n. The modal constant plays a critical role for the type of feedback.

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3. Active vibration control

The purpose of the system for the active vibration control (AVC) is to compensate the effect of a disturbing external force on a vibration of the mechanical structure. The dampening effect of AVC can be assessed by changes in displacement, velocity or acceleration of the selected point of the structure. It can be the free end of the cantilever beam. The result of the active dumping is the minimum motion around the steady-state position and the minimum velocity or acceleration of vibrations. Mechanical structures are usually weakly damped. The analysis shows that for undamped systems there are absent terms of the odd powers of the complex variables *s* in the Laplace transfer function. The undamped system is at the margin of stability. It has been shown that the most appropriate controller for such systems uses a proportional feedback based on the velocity of the controlled displacement (Tůma et al., 2014, 2016). The set point (SP) for such closed-loop system is equal to zero. The gain of the velocity feedback is designated by *T*.

There are two possible solutions, the collocated and non-collocated active vibration control. For the collocated system, the correcting force acts and the response is measured at the same point. For the non-collocated system, it the correcting force acts at the point indexed by q and the vibrations are sensed at the point indexed by r (q ≠ r). An example of the non-collocated system is shown on the right of Fig. 1. The vibration of the free end element of this cantilever beam is sensed at the point $r = 5$, and the correcting force acts at the element just next to the clamped end, therefore $q = 1$.

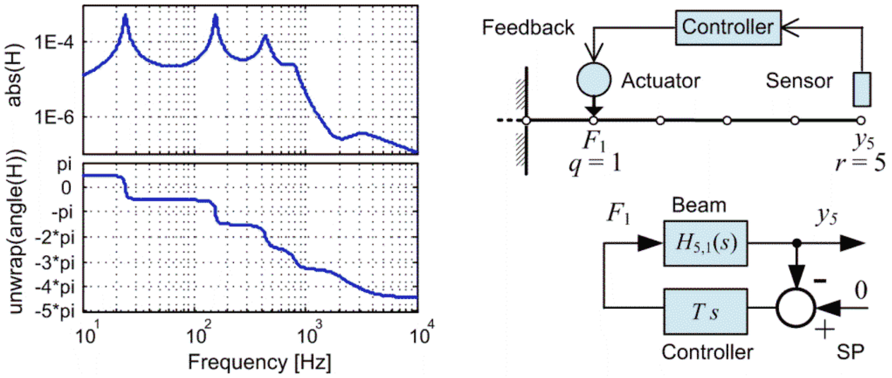

Fig. 1: Frequency response function $j\omega H_{5,1}(\omega)$ and AVC system.

The stability of the feedback system determines the position of the pole of the closed-loop transfer function. The beam is a stable system without control due to the natural damping; it does not start to vibrate by itself. The increase of the feedback gain causes one of the poles that are associated with the two lowest resonant frequencies moves away from the instability margin while the other pole is approaching or exceeding the stability margin.

For the given beam, which is divided into 5 elements, and the assumption of the Rayleigh's damping, the locus of the closed-loop poles are shown in Fig. 2 (Tůma et al., 2014). The root locus demonstrates the effect of the controller time constant *T* change on the system stability. The time constant varies from 0 to 1E6. The poles are calculated as the roots of the polynomial $1 + TsH_{5,1}(s)$.

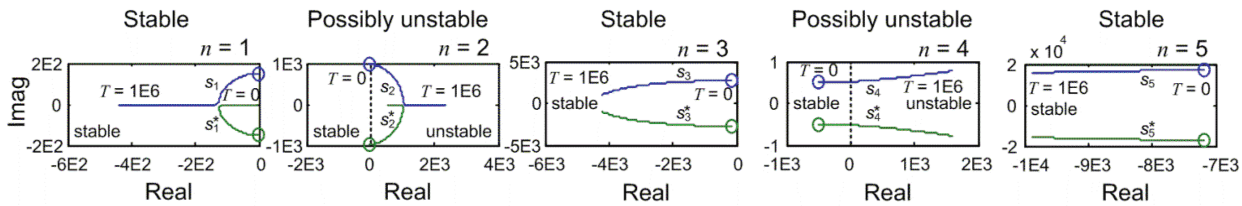

Fig. 2: Root locus to demonstrate the effect of the time constant T change.

Because the degree of polynomial equals to ten in the variable s, the number of roots, i.e. the number of poles is ten as well. Five pairs of poles are complex conjugate. The stability margin crosses the pole for the mode n = 2 and the pole for the mode n = 4 approaches this margin.

The analysis shows that the opposite sign of the modal constants reduces the damping effect of velocity feedback. There are two possible ways how to improve the efficiency of the active vibration control of weakly damped systems

- either control each vibration mode separately
- or change positive feedback to the negative one.

Both the methods indicate the transition of the controller design from the frequency range from zero to infinity to the control in a narrow frequency band (Šuránek et al., 2013, 2014).

The first possible solution with the filter of the band-pass type is shown in Fig. 3. This arrangement of the active vibration control is called a Positive Position Feedback (PPF) controller, see Premont. In the mentioned figure, PPF is intended for two resonant frequencies with the negative and positive feedbacks. The input signal of the filter is a signal of the velocity type because a controller of the proportionality type is used. The velocity signal is obtained by integrating the acceleration signal with respect to time. The acceleration is measured on the free end of the beam as the controlled variable. The integration and band-pass filtering together form a band-pass filter, whose transfer function has been designed previously by many authors (Premont, 1997). The filter is of the second order and therefore causes the least possible delay in the control loop. The output of this band-pass filter is amplified as necessary. The frequency range of the controller is restricted to a narrow band around the resonant frequency.

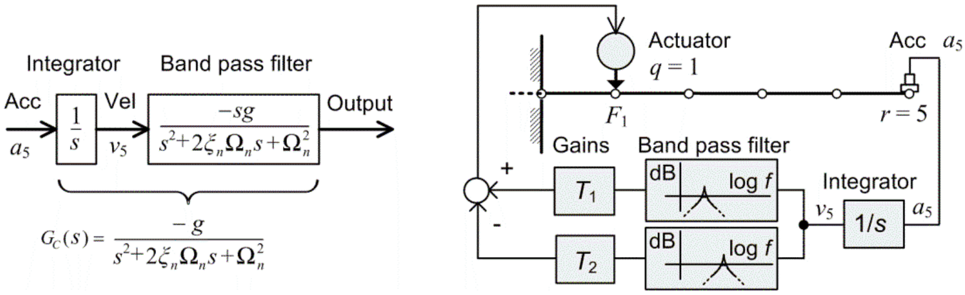

Fig. 3: Principle of Positive Position Feedback (PPF).

The second method for controlling the weakly damped systems is converting the positive feedback to the negative one with the use of an all-pass filter. This type of the frequency filter modifies the phase of the harmonic signal at the output compared to the input without changing the amplitude of the signal frequency components. The filter of this type of the first-order type changes the phase from 0 to π radians in the frequency range from 0 to infinity. The all-pass filter of the second order which transfer function is defined by the formula (2) doubles the phase change. For the possibly unstable modes, it is necessary to change the phase by π at the resonant frequency ω_n of this vibration mode whose modal constant is to be changed from the negative to positive value. The advantage of this filter is the controllable rate of change of phase in comparison to the change of the frequency by setting the value of the relative damping parameter ξ_{APF}.

$$G_{APF,n}(s)=\frac{s^2-2\xi_{APF}\,\omega_n\,s+\omega_n^2}{s^2+2\xi_{APF}\,\omega_n\,s+\omega_n^2} \tag{2}$$

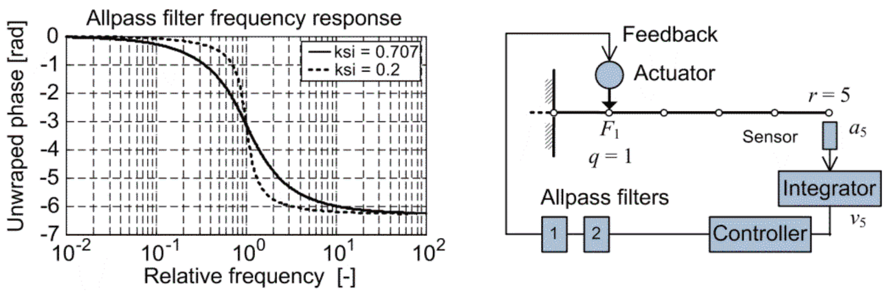

Fig. 4: All-pass filter frequency response (ksi = ξ_{APF}).

Since $\left|G_{APF,n}(j\omega)\right|=1$ only a phase frequency response for two values of the damping parameter ξ_{APF} is shown on the left of Fig. 4. The all-pass filters are connected in series (cascade) with the controller as is shown on the right of Fig. 4. The count of these filters is as many as the count of the negative modal constants. The feedback is of the velocity type and needs an integrator for a sensor of the acceleration type to obtain the velocity signal.

4. Simulation results

The effect of the all-pass filter on the damping of the cantilever beam vibration demonstrates the comparison of the control system response based on the use of two all-pass filters in the closed loop and without them as is shown in Fig. 5. One of the all-pass filters is tuned to the frequency of the second vibration mode and the second one to the frequency of the fourth vibration mode. The beam is excited by a short pulse after 1 second from the beginning of the simulation. During the time delay, the piezo actuator is gradually prestressed. The decaying vibration response without any active vibration control (AVC OFF) is shown on the left of Fig. 5. The effect of ACV without using the all-pass filter (ALL-PASS FILTER OFF) is shown in the middle of Fig. 5. Due to the stability, the open-loop gain can be selected less or equal to 4. The serial connection of two mentioned all-pass filters in the cascade allows increasing the gain of the open-loop in such a way that the time constant T may be increased up to the value of 25 but less than 30 with correspondingly increasing the damping effect as is shown on the right of Fig. 5.

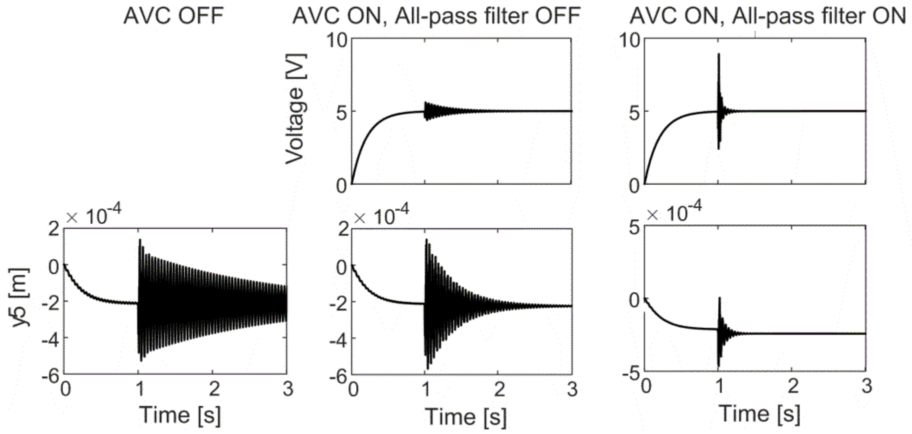

Fig. 5: Decaying vibration without any AVC and two ways of the AVC.

5. Conclusions

The Matlab-Simulink model of the cantilever beam was designed using the method based on the modal analysis. Mechanical systems are generally weakly damped. Stable active damping cannot be designed by classical methods, which were developed for the synthesis of controllers. Some modes for these systems become potentially unstable. The paper describes the method that converts the positive feedback to the negative feedback using the all-pass filter. The effect of the all-pass filter on increasing the damping by shortening the impulse response has been verified by simulation approach.

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