27/28th International Conference
ENGINEERING MECHANICS 2022
Milovy, Czech Republic, May 9 –12, 2022
Paper **#30,** pp. 21–24, doi: 10.21495/51-2-21

STABILITY OF BEAMS UNDER FORCE *F* IN MIDDLE OF SPAN COMBINED WITH END MOMENTS M_a AND M_b

Baláž I.*, Koleková Y., Partov D.*****

Abstract: *Calculation of the critical moments M_{cr} of the beams under various combinations of the force F in the middle of span and the end moments. Solution of the differential equations is presented in the graphical form. Two diagrams enable to obtain the factors C_1 and C_2, which depend on the parameters $\overline{M}$ and ψ. Approximate formula for the calculation of the critical moments depending on three factors C_1, C_2, C_3 and three dimensionless parameters κ_{wt}, ζ_g, ζ_j was derived in (2000). The formula together with the proposed values of factors C_1, C_2, C_3 for basic loadings of beams with the monosymmetric I-sections were accepted for Eurocode (2007) and many national annexes. Presented results are valid for the beams with double symmetric I-sections. They may be used also for the continuous beams being slightly on the safe side. Numerical examples show that our results equal to the exact results of commercial computer programs. The procedure gives quickly practically exact results and therefore it is efficient tool for designers. This paper is twin of the paper (2022) where beams under uniform loading q and end moments are investigated.*

Keywords: Stability of beam in bending, Critical moment, Uniform loading combined with end moments, Double symmetric I-section.

1. Application of proposed procedure

Numerical example 1:

Input values: beam with span L = 6 m simply supported in vertical bending (k_y = 1), horizontal bending (k_z = 1) and torsion (k_w = 1) with rolled section IPE 200 is loaded by force F = 15 kN acting on the surface of the upper flange (z_g = + 100 mm). The values of the end moments are M_a = – 9,6 kNm and M_b = 0 kNm (Fig.1). E = 210 GPa, G = 81 GPa, S 235, f_y = 235 MPa, γ_{M0} = 1,0, γ_{M1} = 1,0. Cross-sectional properties of IPE 200 – DIN 1025, Part 5 (03/1994): h = 200 mm, b = 100 mm, t_f = 8,5 mm, t_w = 5,6 mm, buckling curve "*a*", cross-section Class 1, I_y = 1 943 cm^4, $W_{pl,y}$ = 220,64 cm^3, $M_{pl,y,Rk}$ = 51,85 kNm, I_z = 142,4 cm^4, I_w = 12 746 cm^6, I_t = 6,846 cm^4. For double symmetric section parameter of section symmetry z_j = 0 mm, the factor C_3 is not necessary. $M_{Ed,max}$ = 17,7 kNm is in the middle of the span section x = 3 m.

Numerical example 2:

Input values: the same as in the example 1, only difference is that point load of application of the uniform loading q is on the surface of the bottom flange of IPE 200 section (z_g = – 100 mm). The results of the example 2 are given below in the brackets. Dimensionless parameters of the beam:

$$\kappa_{wt} = \frac{\pi}{k_w L}\sqrt{\frac{EI_w}{GI_t}} = 0{,}364\,,\ \ \zeta_g = \frac{\pi z_g}{k_z L}\sqrt{\frac{EI_z}{GI_t}} = 0{,}385\ (-0{,}385)\,,\ \ \zeta_j = \frac{\pi z_j}{k_z L}\sqrt{\frac{EI_z}{GI_t}} = 0 \qquad (1)$$

a) Calculation of the critical moment M_{cr} of the ideal beam:

Diagram parameters $\overline{M}$ and ψ : $|M_a = -9{,}6\,\text{kNm}| > |M_b = 0\ \text{kNm}| \rightarrow$ $M = M_a = -9{,}6$ kNm, $M_q =$ $= FL/4$ = 15 kN 6 m/4 = 22,5 kNm. For parameters $\psi = M_b / M_a = 0$, $\overline{M} = M/(|M| + M_q) =$ $= -9{,}6/(|-9{,}6| + 22{,}5) = -0{,}299$ we obtain the factors C_1 = 1,46, C_2 = 0,78 from Figs.1, 2.

* Emeritus Prof. Ing. Ivan Baláž, PhD.: Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11; 810 05, Bratislava; Slovak Republic, ivan.balaz@stuba.sk

** Assoc. Prof. Yvona Koleková, PhD., Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11; 810 05, Bratislava; Slovak Republic, yvona.kolekova@stuba.sk

*** Prof. Doncho Partov, PhD.: University of Structural Engineering and Architecture,VSU "Lyuben.Karavelov", Suchodolska 175; 1373 Sofia, Bulgaria, partov@vsu.bg

$$\mu_{cr} = \frac{C_1}{k_z}\left[\sqrt{1+\kappa_{wt}^2+\left(C_2\xi_g+C_3\zeta_j\right)^2}-\left(C_2\xi_g+C_3\zeta_j\right)\right] = 1{,}176,\ (2{,}052) \tag{2}$$

Critical moment: $$M_{cr} = \mu_{cr}\frac{\pi\sqrt{EI_z GI_t}}{L} = 25{,}08\,\text{kNm},\ (43{,}752\ \text{kNm}) \tag{3}$$

Critical moment calculated by LTBeam: $M_{cr,LTBeam} = 24{,}938\,\text{kNm},\ (43{,}391\ \text{kNm})$ (4)

b) Verification of the real beam:

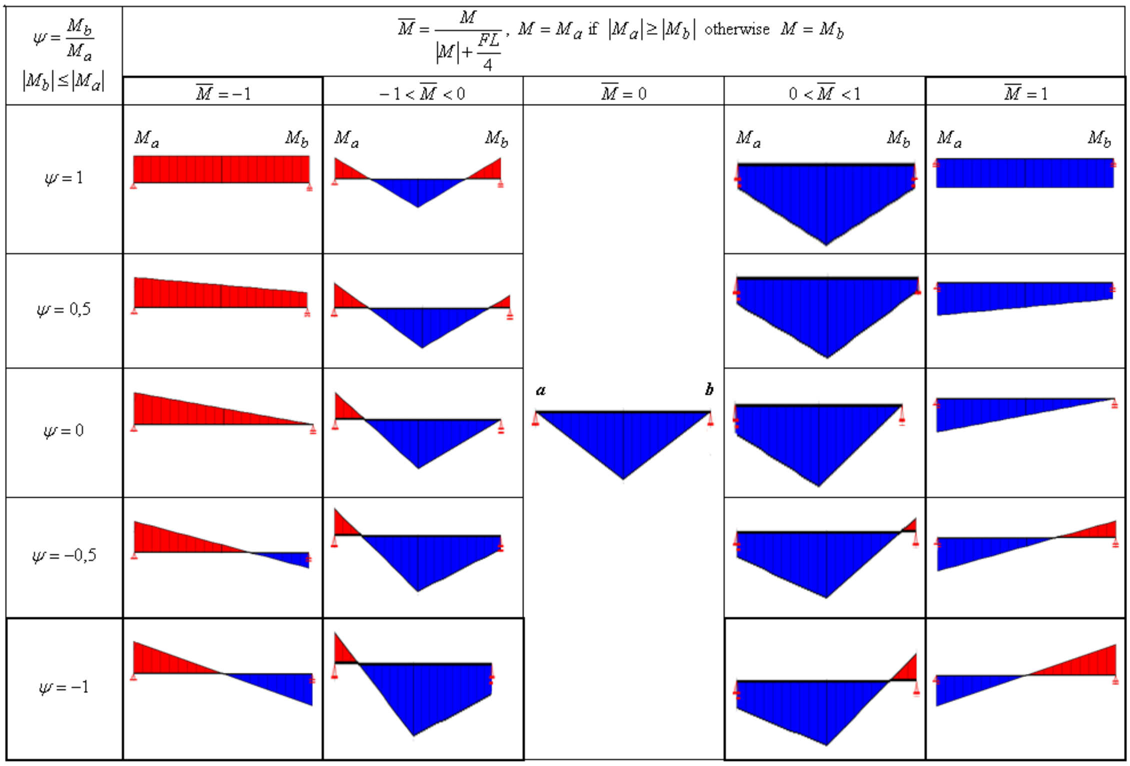
relative slenderness: $\overline{\lambda}_{LT} = \sqrt{\dfrac{W_{pl,y}f_y}{M_{cr}}} = 1{,}438,\ (1{,}089)$, imperfection factor $\alpha_{LT} = 0{,}21$, (5)

$$\Phi_{LT} = 0{,}5\left[1+\alpha_{LT}\left(\overline{\lambda}_{LT}-0{,}2\right)+\overline{\lambda}_{LT}^2\right] = 1{,}664,\ (1{,}186),\quad \chi_{LT} = \frac{1}{\Phi_{LT}+\sqrt{\Phi_{LT}^2-\overline{\lambda}_{LT}^2}} = 0{,}4,\ (0{,}604). \tag{6}$$

Design resistance of the beam: $M_{b,Rd} = \chi_{LT}W_{pl,y}\dfrac{f_y}{\gamma_{M1}} = 20{,}735\ \text{kNm},\ (31{,}308\ \text{kNm}).$ (7)

Utilization factor: $$U = \frac{M_{Ed}}{M_{b,Rd}} = \frac{17{,}7}{20{,}735} = 0{,}854 < 1{,}0,\ (0{,}565<1{,}0). \tag{8}$$

Tab.1: Bending moment distributions as function of the parameters $\overline{M}$ and ψ



Acknowledgement

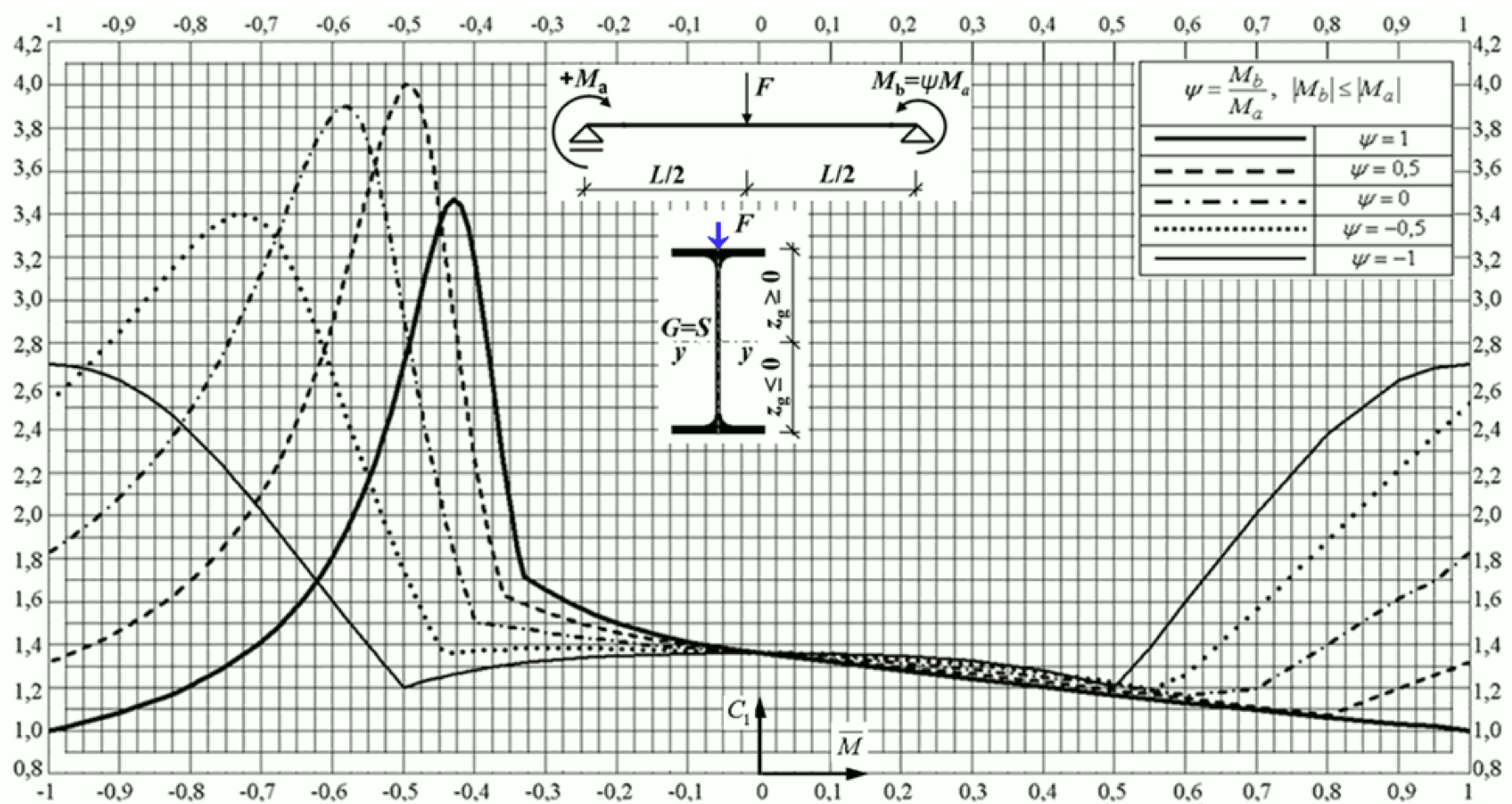
Project No. 1/0244/20 is supported by the Slovak Grant Agency VEGA.

References

Baláž, I., Koleková, Y. (2000) Critical Moments of Beams and Girders. Clark – Mrázik formula. In: *Proc. 19th Czech and Slovak Int. Conf. "Steel Structures and Bridges 2000"*, Štrbské Pleso, pp. 87 – 94.

Baláž, I., Koleková, Y., Partov, D. (2022), Stability of Beams under Uniform Loading *q* Combined with End Moments M_a and M_b. In: Fischer, C. and Náprstek, J., eds., *Engineering Mechanics 2022*, ITAM CAS, Prague, pp. 17 – 20.

EN 1999-1-1 (2007) and Amendment A1 (2009) and Amendment A2 (2013), *Eurocode 9: Design of Aluminium Structures, Part 1.1: General Structural Rules*, CEN Brussels.



$$\overline{M} = \frac{M}{|M| + \frac{FL}{4}}, \quad M = M_a \text{ if } |M_a| \ge |M_b| \text{ otherwise } M = M_b, \quad -1 \le \psi \le 1, \quad -\frac{h}{2} \le z_g \le \frac{h}{2}$$

Fig.1: Factor C_1 for calculation of critical moment M_{cr} of simply supported beam with double symmetric I-section as function of parameters $\overline{M}$ and ψ

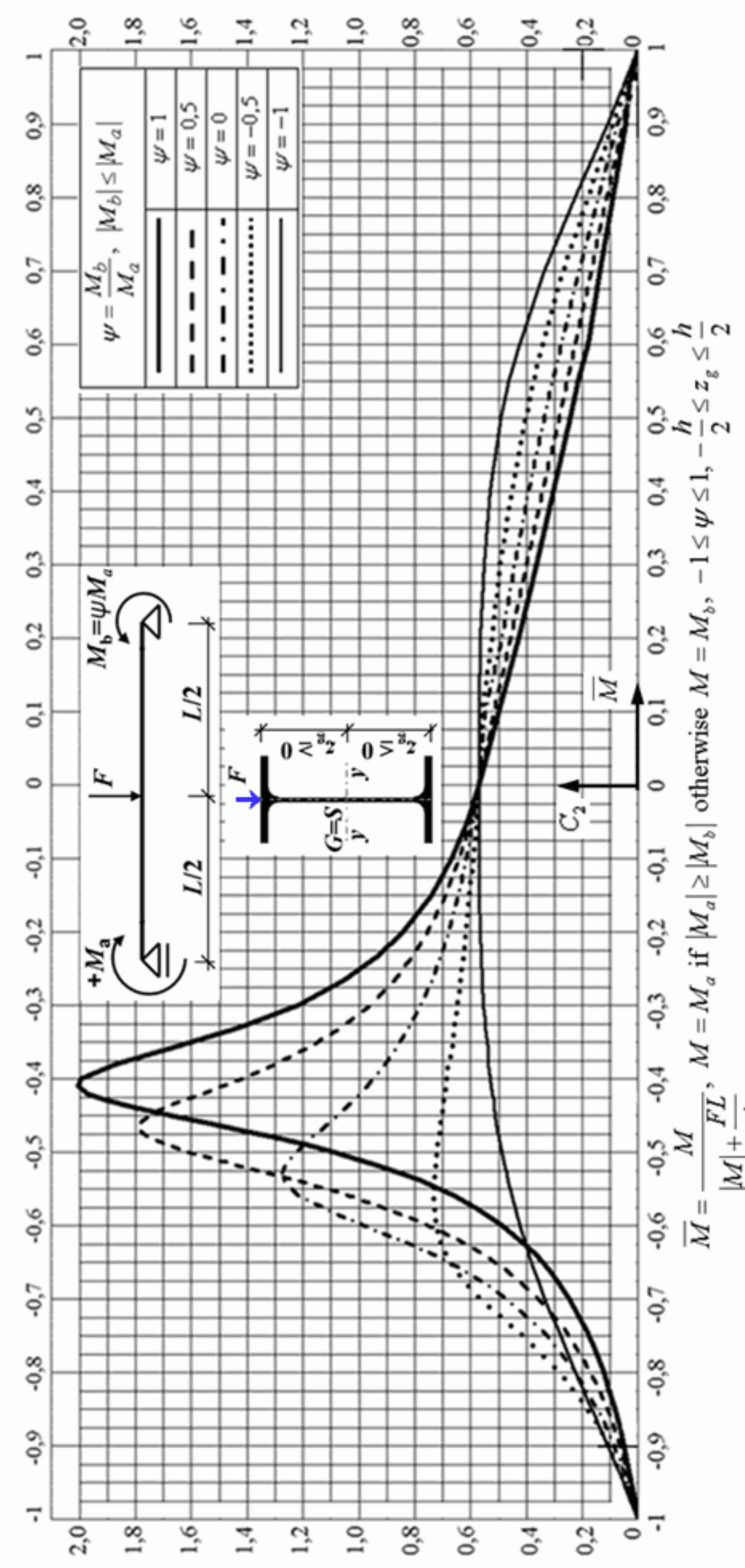


Fig. 2: Factor C_2 for calculation of critical moment M_{cr} of simply supported beam with double symmetric I-section as function of parameters $\overline{M}$ and ψ