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MODELING THE MOVING COGWHEEL LOAD – ANALYSIS USING ANSYS & MATLAB

Bayer J.*

Abstract: *A moving impulse load generated by a heavy cogwheel (CW) can be used as a testing excitation for bridges. This previously proposed type of dynamic testing offers theoretically short testing times. Laboratory experiments confirmed already its capability of damage indication by repeated testing. This contribution suggests an approach how to solve the cogwheel movement using a Finite Element (FE) solver and Matlab. As the solution is theoretically nonlinear, the necessary limiting conditions for its application are formulated. Then the analysis is applied to simulate the passage of the CW over a simple laboratory model in order to compare it to laboratory experiments.*

Keywords: Moving Impulse Load, Modal Analysis, Finite Element Models, From Ansys into Matlab.

1. Introduction

The present research in bridge testing builds on the rich tradition of dynamic increment testing and load testing on the occasion of commissioning of new bridges (Farrar, 1999; Pirner, 2010; Cunha, 2013; Venglár, 2018; Benčat, 2018; Lantsoght, 2019; Cantieni, 1984). High costs for these tests and not entirely useful information obtained from them lad to retreat from this practice. The experimental modal analysis using ambient vibrations is the nowadays most frequently applied alternative, but it is far from a quick and cheap method (Omenzetter, 2013; Zhang, 2005).

A short and reliable testing procedure predestinated for commissioning of bridges as well as for assessment of the condition of the ageing ones is still only a dream for many administrators of traffic roads (Webb, 2015; Commander, 2019).

The testing by a CW is a non-modal approach that offers short testing times and relatively cheap equipment - just a few transducers are needed for the test. So far, a successful indication of a damage was achieved on a model under laboratory conditions (Bayer, 2022). No tests were conducted on real structures, yet. But the idea is challenging and the present results are encouraging.

The analysis of this type of dynamic load is not an elementary task for the following reasons. The inertia relations of the system CW-bridge are changing continuously during the passage. The elasticity of the CW-tips (edges) can cause a sort of jumping of the CW by relatively low velocities. The repeated impact of the CW-tips causes higher harmonic components in the response of the bridge which is an unpleasant phenomenon if we intend to identify structural parameters of the bridge from its response. A reliable analysis is also needed for a model calibration and subsequent health monitoring based on FE model (Simoen, 2015).

Facing these problems, a relatively simple analysis approach was suggested which circumvents the necessity to apply a kinematic model. A standard FE package like ANSYS is used to model the structure and to compute the modal model, but the transient analysis is performed in Matlab according to equations described below under the paragraph Theory.

The Solution for a laboratory model that will be later on used for experiments is presented in the following paragraph. Simulations and the major advantages, limitations and possible applications of the proposed approach are summarized in Conclusions.

* Ing. Jan Bayer, CSc.: Institute of Theoretical and Applied Mechanics of the CAS, Prosecká 809/76; 190 00, Prague 9; CZ, bayer@itam.cas.cz

2. Theory

According to the well-known principles of modal analysis (Ewins,1984), the dynamic behaviour of common building structures can be described with FE models using the following equations (1).

$$\mathrm{M} \cdot \ddot{u} + \mathrm{C} \cdot \dot{u} + \mathrm{K} \cdot u = \mathrm{F} \tag{1}$$

Rewriting it for a coupled system in Figure 1, we obtain the equation (2) (e.g. Rieker, 1996, Lee, 1996)

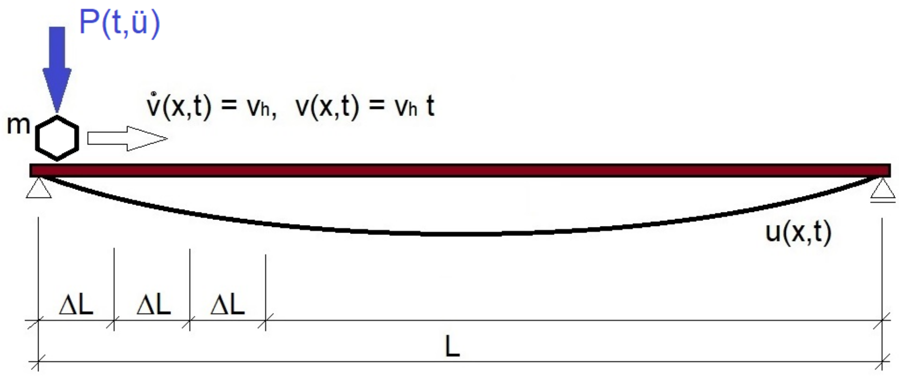

Fig. 1: Schema of the coupled system "Structure – CW"

$$\mathrm{M} \cdot \ddot{u} + \mathrm{C} \cdot \dot{u} + \mathrm{K} \cdot u = \delta_j\big(-m \cdot g - \mathrm{m} \cdot \ddot{u}_j - \mathrm{m} \cdot v_h^2 \cdot u''_j - \mathrm{m} \cdot 2v_h \cdot \dot{u}'_j - F_{st}\big) + P(t, \ddot{u}) \tag{2}$$

$$w = \delta_j \cdot \mathrm{u}_j \ , \tag{3}$$

where the δ_j is the Kronecker delta. The terms on the right hand side represent the mass weight, the inertia force, the centripetal force, the Coriolis force, F_{st} is the force due to the permanent vertical static deformations and $P(t,\ddot{u})$ is the external force generated by a rolling CW with a constant horizontal velocity. When rolling on a firm base the CW would generate a cyclic impulse load with a constant impulse characteristic in each cycle or impulse. But in case it rolls on vibrating structure the vertical impulse force it generates would be dependent on the accelerations $\ddot{u}_j$ of the supporting structure in the sense of Figure 2.

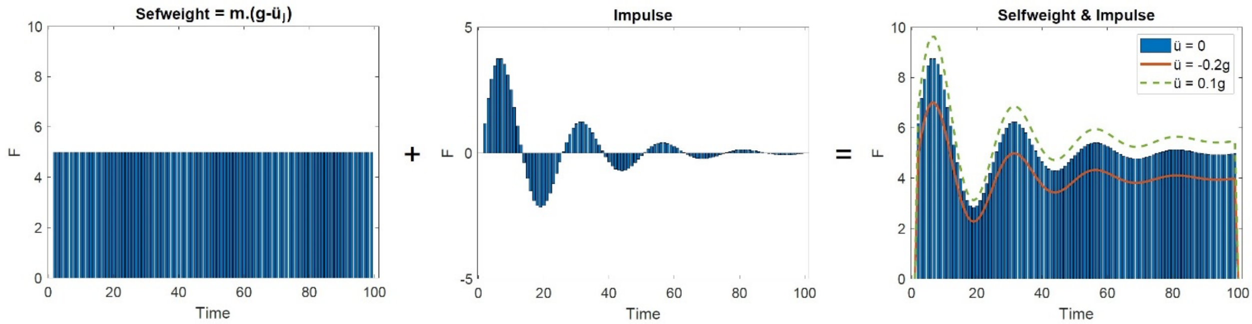

Fig. 2: Assumed loading function caused by the CW with elastic tips / edges

Natural mode shapes of the structure without the moving mass m:

$$\emptyset^T \cdot M \cdot \emptyset = I; \qquad \emptyset^T \cdot C \cdot \emptyset = D; \qquad \emptyset^T \cdot K \cdot \emptyset = \Omega\,. \tag{4a-c}$$

$$u = \emptyset \cdot Q\ ; \qquad \dot{u} = \emptyset \cdot \dot{Q} \qquad \ddot{u} = \emptyset \cdot \ddot{Q}\ ; \tag{5a-c}$$

$$u'' = \frac{d^2\emptyset}{dx^2} \cdot Q\ ; \qquad \dot{u}' = \frac{d\emptyset}{dx} \cdot \dot{Q}\ ; \tag{5d-e}$$

$$Q = [q_1; \dots; q_n]\,, \tag{6}$$

$$\emptyset = [\varphi_{1,1}; \dots; \varphi_{p,n}] \tag{7}$$

and n is the number of modes used and p is a number of finite element nodes. In regard to Fig. 2, the impact force is assumed to be, or is defined as:

$$P(t,\ddot{u}) = f(t) \cdot m \cdot (g - \ddot{u}_j) \tag{8}$$

Rewriting (2) using (3) to (8):

$$I \cdot \ddot{Q} + D \cdot \dot{Q} + \Omega \cdot Q =$$

$$= \emptyset^T \cdot \delta_j \cdot \left(-m \cdot g - m \cdot \emptyset \cdot \ddot{Q} - m \cdot v_h^2 \cdot \frac{d^2\emptyset}{dx^2} \cdot Q - m \cdot 2 \cdot v_h \cdot \frac{d\emptyset}{dx} \cdot \dot{Q} - F_{sw} - f(t) \cdot m \cdot (g + \emptyset \cdot \ddot{Q})\right) .$$

Assuming that $F_{sw} = 0$,

$$I \cdot \ddot{Q} + D \cdot \dot{Q} + \Omega \cdot Q = \emptyset^T \cdot \delta_j m \cdot \left(-(f(t)+1) \cdot (g + \emptyset_{..,j}\ddot{Q}) - 2v_h \frac{d\emptyset_j}{dx}\dot{Q} - v_h^2 \frac{d^2\emptyset_j}{dx^2} Q\right) \tag{9}$$

$$\ddot{u}_j(\mathrm{x},\mathrm{t}) = \sum_{i=1}^{n} \varphi_{i,j} \cdot \ddot{q}_i = \emptyset_{..,j} \cdot \ddot{Q} . \tag{10}$$

After further rearrangements we obtain:

$$\left(I + m \cdot (1 + F_j) \cdot \emptyset_j^T \cdot \emptyset_j\right) \cdot \ddot{Q} + (D + m \cdot \emptyset_j^T \cdot \frac{d\emptyset}{dx}) \cdot \dot{Q} + (\Omega + m \cdot \emptyset_j^T \cdot \frac{d^2\emptyset}{dx^2}) \cdot Q = \delta_j \cdot mg(F_j + 1) \cdot \emptyset_j^T$$

Substituting new symbols $\breve{\mathrm{M}}, \breve{\mathrm{D}}, \breve{\mathrm{K}}$ for matrices on the left hand side we get:

$$\breve{\mathrm{M}} \cdot \ddot{Q} + \breve{\mathrm{D}} \cdot \dot{Q} + \breve{\mathrm{K}} \cdot Q = -\delta_j \cdot mg(F_j + 1) \cdot \emptyset^T . \tag{11}$$

$$w = \delta_j \cdot \emptyset_j \cdot \mathrm{Q} = \mathrm{diag}(\emptyset \cdot Q^T) \tag{12}$$

Only selected degrees of freedom (DOFs) can be imported from the finite element model. But the imported set $\emptyset_a$ has to contain the whole driving path (all the loaded DOFs) and other nodes of interest $\emptyset_b$.

$$\emptyset = \begin{bmatrix} \emptyset_a \\ \emptyset_b \end{bmatrix} . \tag{13}$$

The damping matrix D can be assumed to be proportional and therefore also of the diagonal form

$$\mathrm{D} = \alpha \cdot \mathrm{I} + \beta \cdot \Omega . \tag{14}$$

The evaluation of the contact force F_c in the following equation is essential for the validity of the eq. (1) and the whole analysis.

$$F_c = \delta_j \cdot \left(-m \cdot g \cdot -\mathrm{m} \cdot \ddot{u}_j - m \cdot v_h^2 \cdot u''_j - m \cdot 2 \cdot v_h \cdot \dot{u'}_j - F_{st}\right) + P(t,\ddot{u}) , \tag{15}$$

and in terms of modal coordinates:

$$F_c = -\delta_j \cdot m \cdot \left(\left(1 + f(t)\right) \cdot \left(g + \emptyset_j \cdot \ddot{\mathrm{Q}}\right) + 2 \cdot v_h \cdot \frac{d\emptyset_j}{dx} \cdot \dot{\mathrm{Q}} + v_h^2 \cdot \frac{d^2\emptyset_j}{dx^2} \cdot \mathrm{Q} + F_{st}\right) . \tag{16}$$

A positive F_c indicates that the moving mass had separated from the beam for which the conditions above have not been defined here (Lee, 1996).

3. Analysis

The analytical model corresponds to the experimental model that will be applied later on for experimental verification of the analysis. It is a 4 m long simply supported beam made from acrylic glass. Its H-shaped cross section is 270mm wide and 100mm high glued together of 10 mm thick plates. The beam is stiffened on the bottom part under the carriageway by 10 mm thick transversal ribs and pre-stressed in order to have zero deflection. The total mass of the beam is 23 kg with the first natural frequency of 5.2 Hz. The first 6 bending modes in the frequency band 0-200 Hz was transferred into Matlab. The damping of the model was assumed 3% for all of the modes. The maximum Impulse peak reached 12.3 N and the stiffness of the CW tips was assumed to be XY N/m. The permanent deformations and geometrical imperfection were neglected.

The passage of the 6-edged CW weighting 502 g was modeled with the constant velocity of 2.6 and 3.8 m/s. Time acceleration records were computed in Matlab by HHT-α direct integration and consecutively also the average spectral densities and forced passage shapes were computed from the time records. The obtained numerical results will be presented in the conference.

4. Conclusions

New approach how to simulate a passage of a CW over a bridge was presented. It makes the use of preprocessing and Eigenvalue solution in Ansys which allows for a detailed modeling of any bridge structure. The computed natural modes are then exported into Matlab where the response of the bridge to CW passage is simulated. The simulation requires the function of the contact force under the tip of the CW as the input that can be obtained experimentally for the applied CW. A short solution times are a pleasant feature of the proposed approach.

The application is restricted to relatively low velocities of the CW until the CW starts to jump after the contact with the bridge reducing possible nonlinear phenomena significantly. In this way it circumvents the kinematic formulation that would be otherwise necessary.

The simulations of the CW will be used in the effort to localize a bridge damage from the measured CW response of bridges.

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