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B-SPLINE BASED DENSITY ESTIMATION OF FATIGUE FAILURES IN SLM 18NI300 STEEL

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Abstract: *The lack of probability-fatigue data is still a problem limiting the fatigue optimization of additively manufactured metallic materials. Modelling the probability density of fatigue life random variable is essential to predict the durability of structures. In the paper an experimental data for additively manufactured 18Ni300 maraging steel are studied fitting them to a proposed B-spline model. Parameters of the B-spline basis functions were obtained through a nonparametric density estimation technique. Obtained probability density functions were compared with maximum likelihood estimated Weibull density functions.*

Keywords: MS1 steel, fatigue failure probability, B-spline bases.

1. Introduction

Nowadays, reduction of energy consumption becomes the main engineering issue. The availability of additive manufacturing techniques have greatly increased the potential for mass design (directly connected with the reduction of energy consumption), overcoming the traditional constraints of manufacturing. Within the field of additive manufacturing, Selective Laser Melting (abbrev. SLM, alternatively: DMLS, direct metal laser sintering) becomes one of the most popular processes. In this powder-bed fusion process the desired shape is generated by a high-intensity laser which melts and fuses the metallic powder layer-by-layer. These features have made topology optimization (referred to as fatigue constraints) practical and providing another field of application for fatigue analysis. Topology optimization is an advanced methodology used to generate geometry configurations that are difficult to obtain using conventional processes. Due to the reduced material volume, the optimized geometry should be verified through analysis of the impact of the resulting modified stress tensor components on fatigue reliability. The prediction of fatigue reliability is inseparably linked to analysis of changes in density of fatigue lives at different load levels.

Density of a continuous random variable is a function able to provide a probability that the value of this variable would be equal any sample from the set of possible values. Estimation of probability density function (abbrev. PDF) is being practically applied not only in fatigue life estimation of solids but also in, e.g. reliability analysis. In case of the fatigue phenomena it is reasoned by the fact that fatigue damage process is stochastic in nature (e.g. Sobczyk and Spencer, 2012). The mathematical model behind it is a two-state stochastic process, described by the fatigue life random variable - preferably Weibull distributed in logarithmic space (Blacha and Karolczuk, 2016; Zhao and Liu, 2014; Schijve, 1993). Commonly, the tendency and scatter at the high-cycle fatigue regime are being described by the P-s-n (probability-stress-life) model, which means that scatter (as well as the sample space) is changing and depends on the stress level. It is the reason why the PDF will be shaped differently between each stress level and why probabilistic definition of the s-n field is crucial to ensure a reliable fatigue design of components, in particular for life prediction or failure hazard.

The approaches to density estimation could be divided into parametric and nonparametric. Parametric estimation requires a priori assumption regarding the probability distribution which results in PDF smooth along the entire sample space; an example of a parametric estimation technique is Maximum Likelihoo-

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Estimation (abbrev. MLE). In state of the art in fatigue reliability it can be found as the technique most frequently applied, often combined with the generalized extreme value distribution family of which the Weibull distribution is a part (the type III distribution) (Fernández Canteli et al., 2022). Nonparametric estimation makes minimal assumptions about the underlying distribution but specifically, the resulting PDF could easily become peaked or over-smoothed because the estimation result greatly depends on the number of sample points and span between them.

In the paper an alternative technique is proposed for nonparametric estimation of a smooth PDF for fatigue failure of additively manufactured 18Ni300 steel, combining kernel density estimation and B-spline interpolation bases. The proposed approach allows to define the PDF with relation only to the mean fatigue life and its scatter on the analyzed stress level.

2. Methodology

The paper investigates the possibility of nonparametric density estimation of fatigue failures in 18Ni300 maraging steel using a B-spline interpolation algorithm. In the first step, a MLE algorithm was used to estimate additional PDFs of a log fatigue-life random variable N_l (where $N_l = \lg(N)$ and N is a random number of load cycles to failure). The obtained results were then used to analyze and validate the B-spline PDF's estimated in the next step. The analysis involved fatigue lives corresponding to four stress levels in high-cycle fatigue loading. Entire investigation was based on the results of fatigue tests performed under controlled uniaxial constant-amplitude tension-compression loading, described further in this paper.

2.1. Fatigue tests

The fatigue tests were performed using specimens fabricated by selective laser melting (SLM) on an EOSINT M280 machine at Opole University of Technology, composition of the supplied powder can be found in Tab. 1. The specimens' geometry was designed according to the ASTM guidelines (ASTM E 466-15) and can be seen in Fig. 1.

Element	**Fe**	**Ni**	**Co**	**Mo**	**Ti**	**Al**	**Cr**
Min		17.00	8.50	4.50	0.60	0.05	-
Max		19.00	9.50	5.20	0.80	0.15	0.50
Element	**Cu**	**C**	**Mn**	**Si**	**P**	**S**	
Max	0.50	0.03	0.10	0.10	0.01	0.01	

Tab. 1: Chemical composition of powder used in manufacturing process (wt. %) (EOS, 2022).

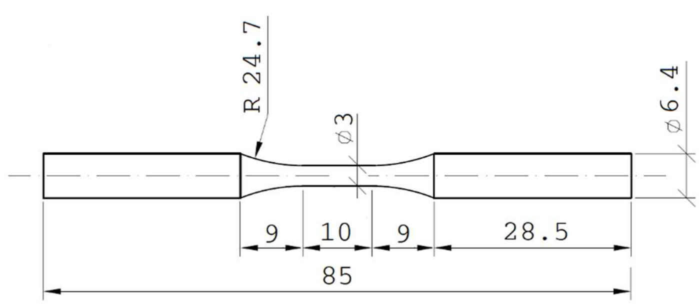

Fig. 1: Geometry of tested specimens.

The specimens were manufactured with a powder-layer thickness of 0.04 mm and 285/138/60 W laser powers for stripe/contour/edge scanning, respectively. During the process, new layers were added in the vertical direction with oxygen concentration in the process gas atmosphere less than 0.25 %. Finished specimens were cut by electrical band saw and heat-treated using a Nabertherm N41/H industrial furnace: maintained at 490 °C for 4 hours and cooled for 48 hours. At the end, surfaces were bead-blasted in order to obtain a smooth finish (glass beads with diameters ∈ <90, 150> µm).

The manufactured specimens were submitted to constant-amplitude uniaxial fully reversed fatigue loading. 20 specimens were tested at each of the σ_a stress amplitudes: 1 200, 1 000 and 800 MPa and 19 specimens were tested at σ_a = 520 MPa. The processed data required to reproduce the findings can be found in data repository (Blacha, 2023).

2.2. Maximum Likelihood Estimation

The test results were used to estimate the parameters of fatigue life distributions in a maximum likelihood estimation approach. PDF of the estimated distributions can be used to formulate and validate the assumptions underlying the proposed B-spline approach.

Maximum likelihood estimation (MLE) is a parametric technique based on an algorithm which evaluates the joint probability density at the observed data sample. The evaluation process determines the local maximum of the likelihood function L, parameterized by a multivariate parameter θ. Here, the likelihood function was represented by the Weibull PDF, assuming the random variable is Weibull distributed, $N_l \sim W(\alpha, \beta)$, where: $N_l = \lg(N)$). In such case, the maximum likelihood estimate $\hat{\theta}$ is a vector whose entries are the values of α and β that make the observed data most probable. Estimates of both distribution parameters, α and β (shape and scale parameter, respectively) can be seen in Tab. 2.

Stress amplitude σ_a, MPa	Shape parameter $\hat{\alpha}$	Scale parameter $\hat{\beta}$
1 200	45.606	3.355
1 000	53.047	3.761
800	59.985	4.215
520	41.868	4.937

Tab. 2: MLE-estimated parameters of Weibull distribution.

2.3. B-spline based density estimation

B-spline interpolation algorithm allows to derive smooth curves and surfaces on the basis of a sequence of limited data which values are known as control points. The curve is fitted in an iterative process with regards to these points and the basis functions N_{ip}, being a function of specific points called knots u, $u = \{u_0, u_1, \ldots, u_m\}$ $(u_{i-1} < u_i)$:

$$N_{i,0}(u) = \begin{cases} 1 & if \quad u_i \le u \le u_{i+1} \\ 0 & else \end{cases}, \tag{1}$$

when $p > 0$:

$$N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u) \tag{2}$$

(where i is the ordinal and p denotes degree of the basis function).

Here it is assumed that PDF of a log-fatigue life can be modeled by a certain basis function with appropriately defined knots. In general, the more knots are defined, the higher degree is assumed and the shape of such function becomes more sigmoidal. Special care must be taken when defining the knot locations, especially the first and the last one (u_0 and u_m, respectively), as it has a major impact on the shape of this function. The undertaken simulations allowed to formulate the knot vector $U = [u_0\ u_1\ u_2\ u_3\ u_4\ u_5\ u_6]$ capable of estimating shape of the PDF for a continuous random variable N_l distributed on each of the tested stress levels. The following knot sequence was assumed and validated:

$$u_1 = \bar{n}_l - 2std,\ u_2 = \bar{n}_l + std,\ u_3 = u_2 + \Delta u,\ \ldots,\ u_6 = u_2 + 4\Delta u \tag{3}$$

It should be mentioned that $U = [u_0 \ldots u_6]$ knot vector results in 6th degree of the applied basis function, i.e. $N_{0,6}$. In the underlying assumption a nonparametric approach for PDF estimation can be used to locate the first and the last knot as well as the Δu interval. Application of kernel density estimation combined with sigmoidal kernel and Silverman's optimum estimate of the smoothing coefficient has provided a reasonable location of the boundary knots. The resulting basis function should be rescaled assuming $u = n_l$:

$$f(n_l) = \frac{N_{0,6}(u)}{\int_0^{\infty} N_{0,6}(u)\, du} \tag{4}$$

In this way, PDF of a log-fatigue life random variable can be obtained at a given load level, in nonparametric approach and on the basis of the mean value and standard deviation.

3. Results and conclusions

Densities obtained at the tested load levels according to the above approaches were compared and illustrated in Fig. 2.

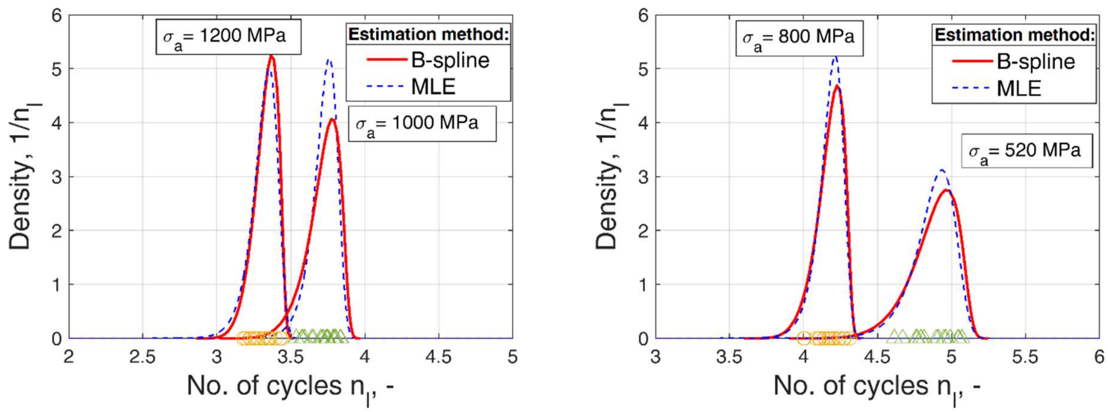

Fig. 2: Comparison between the PDFs estimated according to the proposed B-spline approach and MLE.

In view of Fig. 2 it can be seen that the proposed approach is capable of reflecting the trend in scatter growing along with mean fatigue life. Obtained density functions mostly reflect the derived Weibull densities, biggest discrepancy occurred at the load level where a slight movement outside the trend of stress-life relationship was observed (Blacha, 2023).

The results obtained for SLM-manufactured 18Ni300 steel evidenced the possibility of application of B-spline basis functions to an iterative, nonparametric estimation process of a smooth PDF. The findings can help in estimating a continuous fatigue failure probability distribution in high-cycle fatigue range, at the specified load level without the additional analysis of different load levels.

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References

ASTM E 466-15 *Standard Practice for Conducting Force Controlled Constant Amplitude Axial Fatigue Tests of Metallic Materials*; ASTM International.

Blacha, Ł. (2023) *Fatigue Life for 18Ni300 Steel Under Uniaxial Fully Reversed Loading.* Mendeley Data, V1.

Blacha, Ł. and Karolczuk, A. (2016) Validation of the weakest link approach and the proposed Weibull based probability distribution of failure for fatigue design of steel welded joints. *Engineering Failure Analysis*, 67, pp. 46–62.

EOS Maraging Steel MS1 Material Data Sheet (2022). EU - EOS Store. ms-ms1-m290_material_data_sheet_06-22_en.pdf, 2022 (accessed 13 January 2024).

Fernández Canteli, A., Castillo, E., Blasón, S., Correia, J. A. F. O. and de Jesus, A. M. P. (2022) Generalization of the Weibull probabilistic compatible model to assess fatigue data into three domains: LCF, HCF and VHCF. *International Journal of Fatigue*, 159, art. 106771.

Schijve, J. (1993) A Normal Distribution or a Weibull Distribution for Fatigue Lives. *Fatigue & Fracture of Engineering Materials & Structures*, 16, pp. 851–859.

Sobczyk, K. and Spencer Jr, B. F. (2012) *Random Fatigue: From Data to Theory.* Academic Press, San Diego.

Zhao, Y. X. and Liu, H. B. (2014) Weibull modeling of the probabilistic S–N curves for rolling contact fatigue. *International Journal of Fatigue*, 66, pp. 47–54.