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FREE AXISYMMETRIC VIBRATION OF ELASTICALLY RESTRAINED POROUS ANNULAR PLATE USING HAAR WAVELETS

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Abstract: *Free axisymmetric vibration analysis of a porous annular plate is presented in this paper. Both the edges of the plate are elastically restrained against rotation and translation. The classical plate theory is used to develop the mathematical model. The mechanical properties are varying in thickness direction. The Haar wavelets are used in the analysis. The highest order derivative is approximated by Haar wavelets and a generalized eigenvalue problem is obtained. The first three frequencies for different combinations of restraint parameters, radii ratio and porosity coefficient are obtained. The present analysis is validated by a convergence study. The frequencies for classical boundary conditions are obtained by assuming particular values of restraint parameters and compared with those available in the literature. A close agreement of results is observed.*

Keywords: Axisymmetric, restrained, porous, annular, Haar wavelets.

1. Introduction

Free vibration of structures has been a topic of research for a long time. Porous plates, being lightweight, find applications in many fields. There is a need of research on dynamic behavior of structures made of porous material. This paper considers free axisymmetric vibration of porous annular plate elastically restrained along the inner and outer boundaries. The mechanical properties of the plate material are assumed to be varying along thickness direction. The Haar wavelets are used to calculate first three frequencies. The effects of porosity and restraint coefficients are studied on the frequencies. The results in special cases are compared with those available.

2. Mathematical model

Consider a porous annular plate of uniform thickness h with inner radius b and outer radius a (Fig. 1). The top and bottom surfaces are $z = h/2$ and $z = -h/2$, respectively. The material properties are assumed to be graded in the thickness direction.

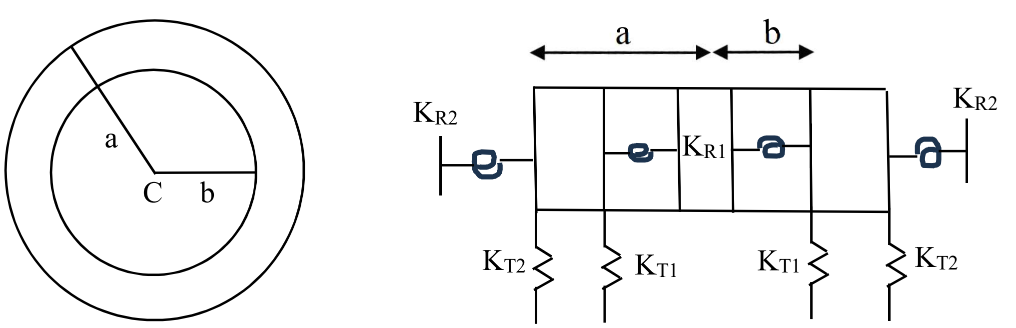

Fig. 1 a) Geometry of the porous annular plate, b) annular plate with constraints at the edges.

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The equation of motion governing axisymmetric vibration of such plate is given as follows (Shariyat and Alipour, 2011):

$$D\frac{\partial^4 w}{\partial r^4}+\left\{\frac{2D}{r}+2\frac{dD}{dr}\right\}\frac{\partial^3 w}{\partial r^3}+\left\{\frac{(2+\upsilon)}{r}\frac{dD}{dr}+\frac{d^2D}{dr^2}-\frac{D}{r^2}\right\}\frac{\partial^2 w}{\partial r^2} +\left\{\frac{D}{r^3}-\left(\frac{1}{r^2}\frac{dD}{dr}-\frac{\upsilon}{r}\frac{d^2D}{dr^2}\right)\right\}\frac{\partial w}{\partial r}+\rho h\frac{\partial^2 w}{\partial t^2}=0, \tag{1}$$

where w is the transverse displacement, $D = Eh^3/12(1-\upsilon^2)$ is the flexural rigidity, υ is the Poisson ratio and ρ is the mass density. For free vibrations, solution of Eq. (1) may be taken as:

$$w(r,t)=\overline{W}(r)\,e^{i\omega t}. \tag{2}$$

Using Eqs. (2) and (1) becomes:

$$D\frac{d^4\overline{W}}{dr^4}+\left\{\frac{2D}{r}+2\frac{dD}{dr}\right\}\frac{d^3\overline{W}}{dr^3}+\left\{\frac{(2+\upsilon)}{r}\frac{dD}{dr}+\frac{d^2D}{dr^2}-\frac{D}{r^2}\right\}\frac{d^2\overline{W}}{dr^2} +\left\{\frac{D}{r^3}-\left(\frac{1}{r^2}\frac{dD}{dr}-\frac{\upsilon}{r}\frac{d^2D}{dr^2}\right)\right\}\frac{d\overline{W}}{dr}-\rho h\omega^2\overline{W}=0. \tag{3}$$

The variation of Young's modulus and mass density per unit volume in the thickness plane is taken as follows:

$$E(z)=E_{max}\left[1-e_1\cos\left\{\frac{\pi(z+0.5h)}{2h}\right\}\right],\ \rho(z)=\rho_{max}\left[1-e_1\cos\left\{\frac{\pi(z+0.5h)}{2h}\right\}\right] \tag{4}$$

where $e_1 = 1-\frac{E_{min}}{E_{max}}$ is the coefficient of plate porosity, defined as void volume to bulk volume ratio, and $0 < e_1 < 1$. The E_{min}, E_{max} are the Young's moduli and ρ_{min}, ρ_{max} are the mass densities at $z = h/2$ and $z=-h/2$, respectively. The Poisson's ratio $\upsilon = 0.3$ is considered to be constant throughout the thickness.

Using the non-dimensional variables $R=\frac{r}{a}, W=\frac{\overline{W}}{h}$ and Eqs. (4) and (3) becomes:

$$A_0\frac{d^4W}{dR^4}+A_1\frac{d^3W}{dR^3}+A_2\frac{d^2W}{dR^2}+A_3\frac{dW}{dR}+A_4W=0, \tag{5}$$

where $A_0 = f(e_1)\,R^3$, $A_1 = 2f(e_1)R^2$, $A_2=-f(e_1)R$, $A_3 = f(e_1)$, $A_4=-\Omega^2\frac{(\pi-2e_1)}{\pi}R^3$,

$$f(e_1)=\frac{\pi^3-6(\pi^2+8\pi-32)e_1}{\pi^3}.$$

By letting $\epsilon = b/a$ and using the transformation $S=(R-\epsilon)/(1-\epsilon)$, the domain $[\epsilon, 1]$ gets converted into the domain [0, 1] of applicability range of Haar wavelets and Eq. (5) takes the following form:

$$B_0\frac{d^4W}{dS^4}+(1-\epsilon)B_1\frac{d^3W}{dS^3}+(1-\epsilon)^2B_2\frac{d^2W}{dS^2}+(1-\epsilon)^3B_3\frac{dW}{dS}+(1-\epsilon)^4B_4W=0, \tag{6}$$

in which $B_0 = f(e_1)\{\epsilon+(1-\epsilon)S\}^3$, $B_1 = 2f(e_1)\{\epsilon+(1-\epsilon)S\}^2$, $B_2=-f(e_1)\{\epsilon+(1-\epsilon)S\}$,

$B_3 = f(e_1)$, $B_4=-\Omega^2\frac{(\pi-2e_1)}{\pi}\{\epsilon+(1-\epsilon)S\}^3$, $D_0=\frac{E_{max}h^3}{12(1-\upsilon^2)}$, $\Omega=\omega a^2\sqrt{\frac{\rho_{max}h}{D_0}}$

and Ω is the frequency parameter.

3. Haar wavelets and integrals

The Haar wavelet transform was proposed by Alfred Haar in 1909. The Haar wavelet is discontinuous and resembles a step function (Hein and Feklistova, 2011). The Haar wavelet family on [0, 1] is defined as

$$h_i(S)=\begin{cases}1 & S\in[S_1,S_2)\\ -1 & S\in[S_2,S_3]\\ 0 & elsewhere\end{cases} \tag{7}$$

where $S_1=\frac{k}{m}$; $S_2=\frac{k+0.5}{m}$; $S=\frac{k+1}{m}$; $m=2^j$; $j=0,1,2,\ldots\ldots,J$ is the scaling factor; $k=0,1,2,\ldots\ldots,m-1$ is the delay factor; $i=m+k+1$. Integer J is the maximal level of resolution.

The first four integrals of the wavelets $h_i(x)$ are (Lepik, 2007):

$$p_{1,i}(S)=\begin{cases}S-S_1 & S\in[S_1,S_2]\\ S_3-S & S\in[S_2,S_3]\\ 0 & elsewhere\end{cases}, \tag{8}$$

$$p_{2,i}(S)=\begin{cases}\frac{(S-S_1)^2}{2} & S\in[S_1,\ S_2]\\ \frac{1}{4m^2}-\frac{(S_3-S)^2}{2} & S\in[S_2,\ S_3]\\ \frac{1}{4m^2} & S\in[S_3,\ 1]\\ 0 & S\in[0,S_1]\end{cases}, \tag{9}$$

$$p_{3,i}(S)=\begin{cases}\frac{(S-S_1)^3}{6} & S\in[S_1,\ S_2]\\ \frac{S-S_2}{4m^2}-\frac{(S_3-S)^3}{6} & S\in[S_2,\ S_3]\\ \frac{S-S_2}{4m^2} & S\in[S_3,\ 1]\\ 0 & S\in[0,S_1]\end{cases} \tag{10}$$

and

$$p_{4,i}(S)=\begin{cases}\frac{(S-S_1)^4}{24} & S\in[S_1,\ S_2]\\ \frac{(S-S_2)^2}{8m^2}-\frac{(S_3-S)^2}{24}+\frac{1}{192m^4} & S\in[S_2,\ S_3]\\ \frac{(S-S_2)^2}{8m^2}+\frac{1}{192m^4} & S\in[S_3,\ 1]\\ 0 & S\in[0,S_1]\end{cases}. \tag{11}$$

The collocation points are defined as

$$S_l=\frac{l-0.5}{2M},\qquad l=1,2,\dots\dots,2M \tag{12}$$

where $M=2^J$.

4. Boundary conditions

Boundary conditions at the edges are given as follows:

at S = 0

$$\epsilon\frac{d^2w}{dS^2}+(1-\epsilon)\left\{\upsilon-\frac{\epsilon K_{R1}}{f(e_1)}\right\}\frac{dw}{dS}=0, \tag{13}$$

$$\epsilon^2\frac{d^3w}{dS^3}+\epsilon(1-\epsilon)\frac{d^2w}{dS^2}-(1-\epsilon)^2\frac{dw}{dS}+\epsilon^2(1-\epsilon)^3\frac{K_{T1}}{f(e_1)}w=0 \tag{14}$$

and at S = 1

$$\frac{d^2w}{dS^2}+(1-\epsilon)\left\{\upsilon+\frac{K_{R2}}{f(e_1)}\right\}\frac{dw}{dS}=0, \tag{15}$$

$$\frac{d^3w}{dS^3}+(1-\epsilon)\frac{d^2w}{dS^2}-(1-\epsilon)^2\frac{dw}{dS}-\ (1-\epsilon)^3\frac{K_{T2}}{f(e_1)}w=0. \tag{16}$$

The Eq. (6) is a fourth order ordinary differential equation with variable coefficients which is solved for using Haar wavelets. According to Chen and Hsiao (2007), the highest order derivative $\frac{d^4W}{dS^4}$ of transverse displacement W is expanded into the Haar series as follows:

$$\frac{d^4W}{dS^4}=\sum_{i=1}^{2M}a_ih_i(S) \tag{17}$$

where a_i are unknown wavelet coefficients.

Integrating (17) four times, it yields that

$$\begin{aligned}\frac{d^3W}{dS^3}&=\sum_{i=1}^{2M}a_i\,p_{1,i}(S)+\frac{d^3W(0)}{dS^3},\\ \frac{d^2W}{dS^2}&=\sum_{i=1}^{2M}a_i\,p_{2,i}(S)+\frac{d^3W(0)}{dS^3}S+\frac{d^2W(0)}{dS^2},\\ \frac{dW}{dS}&=\sum_{i=1}^{2M}a_i\,p_{3,i}(S)+\frac{d^3W(0)}{dS^3}\frac{S^2}{2}+\frac{d^2W(0)}{dS^2}S+\frac{dW(0)}{dS},\end{aligned} \tag{18}$$

and $$W = \sum_{i=1}^{2M} a_i\, p_{4,i}(S) + \frac{d^3W(0)}{dS^3}\frac{S^3}{6} + \frac{d^2W(0)}{dS^2}\frac{S^2}{2} + \frac{dW(0)}{dS} S + W(0).$$

The quantities $\frac{d^3W(0)}{dS^3}, \frac{d^2W(0)}{dS^2}, \frac{dW(0)}{dS}, W(0)$ in Eq. (18) are obtained using boundary conditions (13–16).

5. Results

Expressing displacement function W in terms of Haar wavelets and discretizing Eq. (6) at different grid points, we obtain a generalized eigenvalue problem. This eigenvalue problem is solved for first three frequencies using a computer program developed in MATLAB. The convergence of frequency parameter Ω for different values of various parameters (porosity coefficient e_1 and restraint parameters (K_{R1}, K_{R2}, K_{T1}, K_{T2}) is shown in Tab. 1. Comparison of frequencies of clamped (C) and simply supported (S) annular plate is shown in Tab. 2. A close agreement of results is observed.

	J					
Mode	2	3	4	5	6	7
I	16.001	15.986	15.983	15.982	15.981	15.981
II	32.311	32.263	32.251	32.248	32.247	32.247
III	85.802	85.18	85.029	84.991	84.982	84.979

Tab. 1: Convergence of frequency parameter Ω of elastically restrained porous annular plate for first three modes for $\epsilon = 0.3, e_1 = 0.1$, $K_{R1} = K_{R2} = K_{T1} = K_{T2} = 100$.

				Mode		
Boundary condition	$K_{R1} = K_{R2}$	$K_{T1} = K_{T2}$	**Reference**	**I**	**II**	**III**
Clamped-Clamped	10^9	10^9	Lal and Sharma (2004)	45.3462	125.3621	246.1563
			Present	**45.303**	**125.950**	**246.900**
Supported-Supported	0	10^9	Selmane and Lakis (1999)	21.0790	81.7370	182.54
			Present	**21.107**	**82.064**	**183.08**

Tab. 2: Comparison of frequency parameter Ω of isotropic annular plate for $e_1 = 0, \epsilon = 0.3$.

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