

MODELING OF A QUARTER-CAR MODEL PASSING AN OBSTACLE USING LAPLACE TRANSFORMS, MATLAB/SIMULINK AND SIMSCAPE

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Abstract: *In this paper, the possibilities of modelling a quarter model of a car as it passes over an obstacle are presented. The Laplace transform was first used to solve the differential equations that describe this dynamic action. As the next option the solution of the derived differential equations is using Matlab/Simulink and finally the results were verified using a model that was created in the Matlab/Simulink/Simscape environment.*

Keywords: Laplace transform, dynamics modelling, Matlab, Simscape, physical modelling.

1. Introduction

Knowledge from practice is that a large group of problems solved in engineering practice is related to the solution of linear differential equations. These can be used to describe the amount of money in a savings bank, the orbit of a spacecraft, the amount of deformation of elastic structures, the description of radio waves, the size of a biological population, the current or voltage in an electrical circuit, etc. In fact, differential equations can be used to explain and predict new facts about anything that is constantly changing. In more complex systems, we do not use just one differential equation, but a system of differential equations, as in the case of an electrical network of multiple circuits or in a chemical reaction with several interacting chemicals. Differential equations with constant coefficients can then be solved using the Laplace transform to obtain their solution, which is a mathematical description of dynamical systems, see Yang (2017) and Harris (2014). In the present paper, the procedure of solving differential equations using Laplace transform is described on a quarter car model, Kmec (2022). In the following section, a solution to the same problem is then shown using modern simulation techniques. In the first case, a differential equation model was created in the Matlab/Simulink environment, which was then compared with the model created in the Matlab/Simulink/Simscape environment, Dvorak (2023) and Slough (2021).

2. Laplace transform

Solving differential equations often encounters problems, especially in terms of time. Integral transformations can be used to solve differential equations with constant coefficients in certain cases, Hroncova (2012). Equations defined in the time domain can be modified into qualitatively different

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equations, determined by so-called pairs of the original equations in a completely different domain, e.g. the frequency domain. The mathematical manipulation of the pairs is then considerably easier. Solving in this domain is finding the pair of the solution of a differential equation. After determining the search variables in the domain of their pairs, we adjust them in the opposite way again to the original time domain and obtain the resulting solution. The transformation of differential equations (originals) into algebraic equations (pairs), that is, the transformation is very convenient, since whole large dictionaries of originals and their corresponding pairs are available. The Laplace one-sided transform is a functional integral transform, introduced by Laplace and defined for $t \geq 0$ by the relation:

$$F(s) = L\{f(t)\} = \int_0^{\infty} f(t) e^{-st} dt, \quad (1)$$

where: L is the Laplace operator, $f(t)$ is a function of time which is called the original, $F(s)$ is the Laplace pair of the original, s is the argument of the Laplace transform, it is a complex variable.

3. Vehicle crossing an obstacle

A quarter car model will be considered, which is constructed from two masses M , which represents the mass of the sprung body section, and mass m , which represents the mass of the axle, wheel and tire (Fig. 1). The constant B is the viscous damping constant, K is the spring stiffness constant of the main suspension and k is the tire stiffness constant. The deflections $y_1(t)$ and $y_2(t)$ represent the displacements from the equilibrium positions.

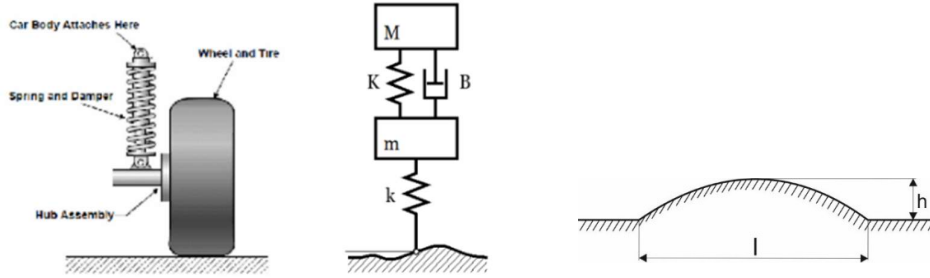


Fig. 1: Dynamic model of the vehicle and the obstacle.

The roughness of the terrain can be described by the equation:

$$s = s(\lambda), \quad (2)$$

where λ is the vehicle path at constant speed.

The equations of motion of a dynamic system can be written in the form:

$$m\ddot{y}_1 + B y_1 - B y_2 + (K + k)y_1 - K y_2 = ks(\lambda), \quad (3a)$$

$$M\ddot{y}_2 + B y_2 - B y_1 - K y_1 + K y_2 = 0. \quad (3b)$$

The obstacle equation is:

$$s = h \sin \frac{\pi}{l} \lambda \quad \text{pre } \lambda \in \langle 0,1 \rangle. \quad (4)$$

In the next step, damping B will not be considered, so the equations of the dynamical system will be:

$$m\ddot{y}_1 + (K + k)y_1 - K y_2 = ks(\lambda), \quad (5a)$$

$$M\ddot{y}_2 - K y_1 + K y_2 = 0. \quad (5b)$$

The partial solutions of these equations are assumed to be of the form:

$$y_j = a_j \sin(\Omega t + \gamma), \quad (6)$$

where a_j are the amplitudes of the displacements y_1 and y_2 from the equilibrium position. After substituting (6) into Eqs. (5a) and (5b) and adjusting:

$$[(k_1 + k_2) - m_1 \Omega^2] a_1 - k_2 a_2 = ks(\lambda), \quad (7a)$$

$$-k_2 a_1 + (k_2 - m_2 \Omega^2) a_2 = 0. \quad (7b)$$

Solving these equations using the determinant yields a frequency equation for calculating the natural angular frequencies Ω_1 and Ω_2 :

$$\Omega_{1,2} = \sqrt{\frac{[m_1 k_2 + m_2 (k_1 + k_2)] \pm \sqrt{[m_1 k_1 + m_2 (k_1 + k_2)]^2 - 4 m_1 m_2 k_1 k_2}}{2 m_1 m_2}} \quad (8)$$

Laplace transform of the equations of the dynamical system (5) under zero initial conditions:

$$m s^2 Y_1(s) + (K + k) Y_1(s) - K Y_2(s) = k \frac{h}{s}, \quad (9a)$$

$$M s^2 Y_2(s) - K Y_1(s) + K Y_2(s) = 0. \quad (9b)$$

Transfer functions are:

$$Y_1(s) = \frac{kh(K + Ms^2)}{s[Mm(s^2 + \Omega_1^2)(s^2 + \Omega_2^2)]}, \quad Y_2(s) = \frac{kK}{s[Mm(s^2 + \Omega_1^2)(s^2 + \Omega_2^2)]}. \quad (10)$$

The inverse transformation is performed by decomposing the functions $Y_1(s)$ and $Y_2(s)$:

$$Y_1(s) = \frac{kh(K + Ms^2)}{s[Mm(s^2 + \Omega_1^2)(s^2 + \Omega_2^2)]} = \frac{kh}{Mm} \left[\frac{K}{\Omega_1^2 \Omega_2^2} \frac{1}{s} + \frac{K - M \Omega_1^2}{\Omega_1^2 (\Omega_1^2 - \Omega_2^2)} \frac{s}{s^2 + \Omega_1^2} + \frac{M \Omega_1^2 - K}{\Omega_2^2 (\Omega_1^2 - \Omega_2^2)} \frac{s}{s^2 + \Omega_2^2} \right], \quad (11a)$$

$$Y_2(s) = \frac{kK}{s[Mm(s^2 + \Omega_1^2)(s^2 + \Omega_2^2)]} = \frac{kKh}{Mm} \left[\frac{1}{\Omega_1^2 \Omega_2^2} \frac{1}{s} + \frac{1}{\Omega_1^2 (\Omega_1^2 - \Omega_2^2)} \frac{s}{s^2 + \Omega_1^2} + \frac{1}{\Omega_2^2 (\Omega_1^2 - \Omega_2^2)} \frac{s}{s^2 + \Omega_2^2} \right]. \quad (11b)$$

Inverse Laplace transform of Eqs. (11a) and (11b):

$$y_1(t) = \frac{kh}{Mm} \left[\frac{K}{\Omega_1^2 \Omega_2^2} + \frac{K - M \Omega_1^2}{\Omega_1^2 (\Omega_1^2 - \Omega_2^2)} \cos \Omega_1(t) + \frac{M \Omega_1^2 - K}{\Omega_2^2 (\Omega_1^2 - \Omega_2^2)} \cos \Omega_2(t) \right], \quad (12a)$$

$$y_2(t) = \frac{kK}{Mm} \left[\frac{1}{\Omega_1^2 \Omega_2^2} + \frac{1}{\Omega_1^2 (\Omega_1^2 - \Omega_2^2)} \cos \Omega_1(t) + \frac{1}{\Omega_2^2 (\Omega_1^2 - \Omega_2^2)} \cos \Omega_2(t) \right]. \quad (12b)$$

The solution of the differential equations of the quarter car model using the Laplace transform was then verified using Matlab, where two methods were used:

- solving the differential equations using their models in Simulink - in this method it is necessary to know the differential equations describing the problem to be solved (Fig. 2),
- in the second case, the Model Based Design approach was used using the Simscape superstructure, where physical modelling is involved and in this case, it is not necessary to know the differential equations describing the problem to be solved (Fig. 2).

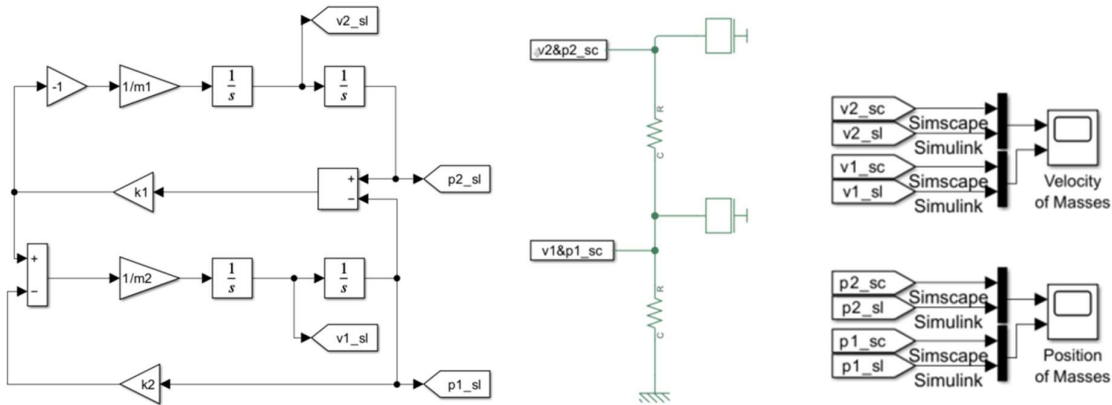


Fig. 2: Simulink model of the differential equation and physical model of vehicle.

In the following figures showing the velocities and positions of the two masses, you can see the correspondence between the solution of the differential equations in the Simulink environment compared to the solution using the physical modelling environment in Simscape (Fig. 3).

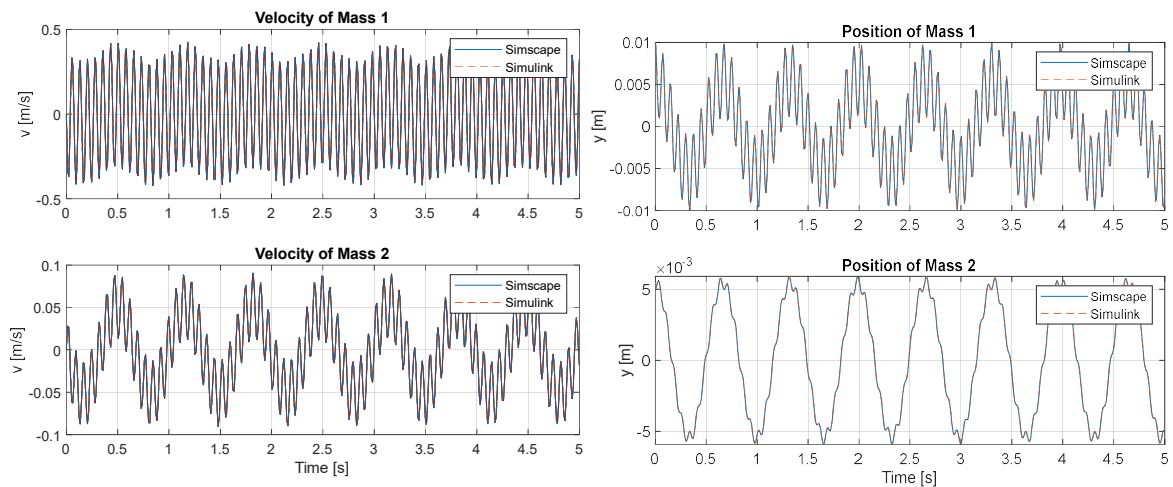


Fig. 3: Final velocity and position of the mass 1 and 2.

4. Conclusions

In this paper, a method for solving differential equations using the Laplace transform and using modern simulation methods is presented. These approaches have been applied to the example of solving a vehicle passing over an obstacle. A quarter car model was created and differential equations were developed for this case. The solution using Laplace transform represents a classical solution to such a problem, which is time consuming and presents a higher error generation rate. Subsequently, two procedures for solving the differential equations were implemented using Matlab. The first method requires knowledge of the differential equations describing the problem to be solved, which can be problematic in some cases. The second method used consists of physical modelling, where the main advantage is that we do not need to know the differential equations and the resulting model consists of real components. Such a solution represents a simpler way of solving the problem, especially when it is necessary to solve more complex systems, in which it is often complicated to set up and then solve differential equations.

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