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# ELASTOPLASTIC MODELS FOR INTERPRETING INDENTATION RESULTS

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**Abstract:** *One of the current and widely used non-destructive testing methods for monitoring and determining the elastic properties of materials is indentation. For interpretation of the test results, a non-trivial task of constructing an adequate mathematical model of the indentation process arises. In numerous cases, analytical formulas are used that are obtained from an elastic linear formulation of problems on the indentation of a non-deformable punch into a homogeneous elastic half-space. Currently, the numerical formulation of the problem makes it possible to obtain and use a numerical solution obtained taking into account the complete plastic nonlinear behavior of the material. In this work, a study of contact problems on the introduction of a spherical and conical indenter into an elastoplastic homogeneous half-space was carried out. To verify the numerical solution, the problem of introducing a spherical and conical indenter into an elastic homogeneous half-space was also solved and compared with known analytical solutions. Issues of convergence and tuning of numerical methods, the influence of plasticity and the applicability of analytical solutions were explored. Problems were solved numerically using the finite element method in the Ansys Mechanical software package.*

**Keywords: Continuous contact, contact mechanic, contact problem, indentation, conical indenter, spherical indenter, finite element method.**

## 1. Introduction

Indentation is used for non-destructive testing of materials and obtaining mechanical characteristics (Bulychev and Alekhin, 1990; Golovin, 2009): hardness, elastic properties of bulk materials and coatings, etc. The essence of the method is to press a more rigid punch, called indenter (usually made of diamond or hard alloys) into the surface of the test sample and obtain diagrams of force depending on the indentation depth on a nanometer scale. The elastic properties of materials and coatings under study are determined from the analysis of the force-displacement diagram at the unloading stage. The Field-Swain (spherical indenter) (Field and Swain, 1993) and Oliver-Pharr (Berkovich indenter) (Pharr and Oliver, 1992) methods are based on solutions to contact problems of theory of elasticity for spherical and parabolic punches (Hertz, 1881; Johnson, 1989). In (El-Sherbiney and Halling, 1996; Sadyrin et al., 2020 and Vasiliev et al., 2020), approximate analytical solutions of axisymmetric contact problems on indentation of a spherical, conical and cylindrical punch into an elastic half-space with a functionally graded coating were constructed.

## 2. Methods

The hypothesis of small indenter movements used in the presented works imposes serious restrictions on the use of analytical applied formulas, since even a small indenter displacement causes the development of plastic deformations. The indentation force and contact area differ significantly from the analytical results.

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The present paper examines the problems of indentation of spherical and conical indenters into an elastoplastic half-space, in a static axisymmetric statement. To implement plastic deformation, a bilinear material model with an elastic modulus (Young's modulus) and a tangential elastic modulus was used. The indenter is made of elastic diamond. The half-space is made of aluminum. The Young's modules of the indenter and half-space material were 1 000 GPa and 70 GPa, respectively. The yield strength and tangential elastic modulus for the bilinear half-space material were 0.28 GPa and 0.5 GPa, respectively.

Contact problems are nonlinear problems due to the changing status of the contact and the stiffness matrix, and require special attention to the accuracy and convergence of the solution. Below are the settings of the numerical methods used in the Ansys finite element analysis package and applied in this calculation.

To implement the contact problem, the "Augmented Lagrange" contact algorithm was used. This is a modified contact algorithm of the common "Pure Penalty" method ("penalty function method"), characterized by the presence of an additional term λ in the expression of the contact force:

$$F_n = k_n \cdot x_p + \lambda, \tag{1}$$

The value of the contact stiffness $k_n$ has a major influence on accuracy and convergence. A large value of stiffness provides high accuracy, but degrades convergence and vice versa. Using the additional term λ allows one to reduce the sensitivity of the algorithm to the contact stiffness $k_n$, and allows one to obtain acceptable results with the value $k_n = 1$, but also requires a larger number of iterations. To "recognize" contacts, the "Gauss point detection" method was used, in which additional points were added on the edges of the elements. To improve convergence, "Normal from Contact" recognition was used with an increase in the number of calculations.

In the present work, to construct a finite element mesh, an 8-node element PLANE183 was used - a high-order element with intermediate nodes. In the area of contacts, the meshes were refined. For indenters, a larger mesh was used. In this case, the reference parameters, relative to which the mesh dimension and linear dimensions of the half-space should be set, are the contact area (indentation depth). Thus, the used partition, for example, in the elastic problem provided about 30 elements in the contact area for a spherical indenter, and 10 elements for a conical one.

For the problem of a spherical punch indentation, refinement of the mesh will obviously lead to more accurate results, and the problem will converge. For the cone indentation problem, setting up the mesh was a much more difficult task because a singularity was formed at the center of the cone. In other words, refinement of the mesh in the center of the cone lead to a direct increase in stress and divergence of the problem. Also, for the problem with a conical indenter, the shape of the elements in the center of the contact was quite important - the shape and size of the elements should have provided greater deformation and prevented the cells from "collapsing". One way to deal with numerical singularity is to create a rounding at the tip of the cone, or to use plasticity models.

For contact surfaces, elements such as CONTA172 and TARGE169 were used with automatic recognition and limitation of the contact area. Moreover, since the indenter rigidity was several times greater than the rigidity of the indented material, the TARGE169 elements were applied specifically to the indenter. Ansys also allows one to take into account geometric nonlinearity for large deformations by including in the calculation the nonlinear strain tensor $\boldsymbol{\varepsilon}(x, y, z)$ with respect to the derivative displacements $\boldsymbol{u}'(x, y, z)$ ("Large deflection"). Thus, in contrast to analytical theories, where only contact nonlinearity was taken into account under the hypothesis of small deformations, we were able to numerically realize all three types of nonlinearity: contact, geometric (large deformations) and physical (plasticity) ones.

Analytical results for comparison were taken from the well-known formulas (2)–(8) for the indentation of rigid punches (Johnson, 1989):

$$a_s = (Rd)^{\frac{1}{2}}, \tag{2}$$

$$a_c = \frac{2d}{\pi \cdot \tan(\varphi)}, \tag{3}$$

$$F_s = \frac{4}{3} E^* R^{\frac{1}{2}} d^{\frac{3}{2}}, \tag{4}$$

$$F_c = \frac{2}{\pi} E \cdot \frac{d^2}{\tan(\varphi)}, \tag{5}$$

$$p_s = p_0 \left(1 - \frac{r^2}{a^2}\right)^{\frac{1}{2}},\ p_0 = \frac{2}{\pi} E^* \left(\frac{d}{R}\right)^{\frac{1}{2}}, \tag{6}$$

$$p_c = \frac{Ed}{(1-\nu^2)\pi a} \ln\left(\frac{a}{r} - \left(\left(\frac{a}{r}\right)^2 - 1\right)^{\frac{1}{2}}\right), \tag{7}$$

$$E^* = \frac{E}{1-\nu^2}. \tag{8}$$

To take into account the rigidity of a diamond spherical indenter in expressions (2)–(8) for effective rigidity $E^*$, the following formula was used:

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}. \tag{9}$$

Here $\boldsymbol{a}$ is the contact radius, $\boldsymbol{d}$ is the indentation depth, $\boldsymbol{\varphi}$ is the angle between the horizontal and lateral planes of the cone, $\boldsymbol{r}$ is the vertical coordinate, $\boldsymbol{E_1}, \boldsymbol{E_2}$ and $\boldsymbol{\nu_1}, \boldsymbol{\nu_2}$ are Young's moduli and Poisson's ratios of the indenter and half-space, respectively, $\boldsymbol{F}$ is the vertical force, $\boldsymbol{p}$ – pressure in the contact area. The indices *s* and *c* here and below in the graphs indicate membership in a sphere and a cone, respectively. The error increased with increasing deformation and for maximum values was about 2 % for the vertical force.

## 3. Results

Figs. 1 and 2 show the results for spherical and conical indentation into an elastic aluminum half-space. It is worth noting that these loads for the elastic problem significantly exceed the permissible ones in terms of the yield strength and were considered as model problems for comparing models. Within the limits of elastic deformations, the error in the numerical results was less than 0.1 %. When solving an elastoplastic problem numerically, we observed linear unloading area, which was used for estimation of Young's modulus using non-destructive testing methods (nanoindentation).

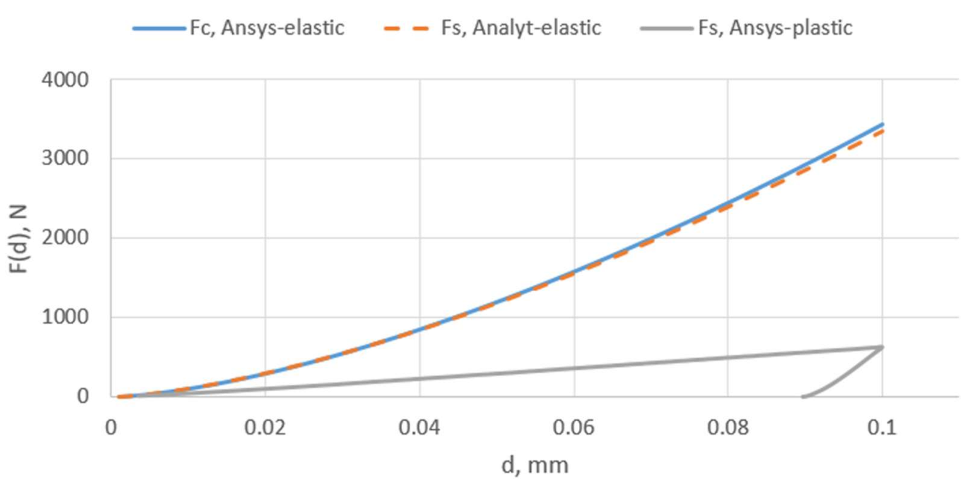


*Fig. 1: Vertical force from displacement for a spherical indenter d.*

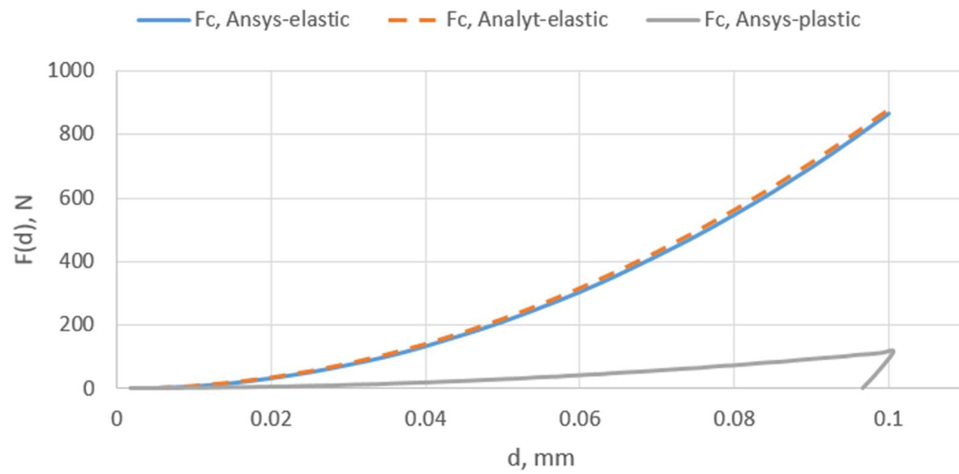


*Fig. 2: Vertical force from displacement for a conical indenter d.*

## 4. Conclusions

The contact area for the elastic model and the elastoplastic model was demonstrated to be significantly different. During plasticity stage, the material was “squeezed out” from under the indenters and significantly increased the contact area. For a spherical indenter for maximum displacement, the difference in contact radius was about 30 %, for a conical indenter – 45 %. In general, the use of an elastic model can serve as a model problem for verification and calibration of numerical methods. However, for large deformations, it is recommended to use an elastoplastic deformation model, and also a nonlinear strain tensor model (large deformation model). These studies can effectively be used to evaluate the accuracy and analysis of models used in identifying the properties of graded, multilayer and coated materials.


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