

## **NON-LINEAR PROBABILISTIC ASSESSMENT OF EXISTING BRIDGE CONSIDERING DETERIORATION**

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**Abstract:** *The integration of the non-linear finite element method with deterioration-based analysis represents a powerful approach for accurately simulating structural behaviors. However, conducting fully probabilistic analyses on large-scale models that incorporate numerous stochastic variables remains a significantly time-intensive task. As a result, structural designers and engineers frequently prefer semi-probabilistic methods, which significantly reduce the need for extensive non-linear computations. This paper concentrates on evaluating the reliability of an existing railway bridge constructed with cast-in steel beams. It employs an advanced numerical analysis through a non-linear finite element model, combined with established semi-probabilistic approaches, including the ECoV method and the newer eigen ECoV method. To accurately reflect the current state of the bridge's superstructure, the analysis models the carbonation of concrete and the ensuing corrosion of steel beams using analytical models that consider both the fundamental material properties and environmental factors.*

**Keywords:** **Non-linear finite element analysis, stochastic analysis, deterioration-based analysis, reliability analysis, semi-probabilistic methods.**

### **1. Introduction**

Assessing the residual carrying capacity of bridges is crucial for civil engineers to ensure the safety of road and railway infrastructure. Utilizing reliability assessment methods alongside finite element method (FEM) analysis, which accounts for material and geometric non-linearities, allows for a more realistic simulation of structural behavior. This approach offers a precise evaluation of existing structures. According to EN 1994-2 (2005), non-linear analysis can be employed to evaluate the global resistance of bridges, though specific application rules are not provided. Despite the high accuracy of non-linear FEM, incorporating uncertainty in material parameters is vital for accurate real-world modeling, making complex stochastic analysis necessary over deterministic approaches. The structural condition and degradation processes, like concrete carbonation or corrosion of metal components, significantly affect the load capacity assessment, underscoring the importance of advanced stochastic non-linear numerical analysis in evaluating bridge deterioration. This comprehensive assessment is crucial for allocating maintenance budgets efficiently. The methodology for this advanced deterioration-based probabilistic assessment, applicable to both reinforced and prestressed concrete as well as steel-concrete structures, is detailed in research by Červenka (2008) and Šomodíková et al. (2016). The paper presents an analysis of a standard railway bridge in the Czech Republic, determining its load-carrying capacity through probabilistic and semi-probabilistic methods combined with a FEM model, and considers the current state and degradation of concrete, reinforcement, and steel elements, modeling the effects of concrete carbonation and steel corrosion.

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## 2. Methods

In case of reliability assessment of existing structures based on the non-linear FEM analysis semi-probabilistic methods are being increasingly used due to the time-consuming nature of FEM simulation. In general, a structure is reliable if the structural resistance,  $R$ , is greater than the action effect,  $E$ . Failure of the structure is represented by the condition  $R - E < 0$  with  $Z = R - E$  being the so-called safety margin. In the semi-probabilistic approach, the resistance of structure,  $R$ , is separated and the design value,  $R_d$ , that satisfies safety requirements is evaluated. For this purpose, the ECoV method (Červenka, 2008) and the eigen ECoV method (Novák & Novák, 2021) are used in this paper.

### 2.1. ECoV method

The ECoV method by Červenka (2008) works with the estimation of the coefficient of variation (CoV) of the resistance,  $v_R$ , defined based on the estimation of the mean and characteristic values of the resistance ( $R_m$  and  $R_k$ ). Assuming a log-normal distribution of the resistance, the coefficient of variation is defined as:

$$v_R = \frac{1}{1.645} \ln \left( \frac{R_m}{R_k} \right) \quad (1)$$

When using  $v_R$ , the global safety factor is calculated according to the formula:

$$\gamma_R = \exp(\alpha_R \beta v_R) \quad (2)$$

where  $\alpha_R$  is the sensitivity factor defined by the value of  $\alpha_R = 0.8$  according to EN 1990 (2002) and  $\beta$  is the reliability index defined for a given design situation. The design value of the resistance is calculated as:

$$R_d = \frac{R_m}{\gamma_R} \quad (3)$$

### 2.2. Eigen ECoV method

A proposed eigen ECoV method by Novák and Novák (2021) is derived directly from Taylor series expansion, a classical method for a statistical analysis of function of random input vector. However, there is an assumption of fully correlated input random variables similarly to ECoV according to the fib Model Code 2010 (2013). The eigen ECoV formula for the estimation of  $v_R$  is defined in the following form:

$$v_R \approx \frac{3R_m^{-4} R_{\theta \Delta} + R_{\theta \Delta}}{\Delta_{\theta}} \cdot \frac{\sqrt{\lambda_1}}{R_m} \quad (4)$$

With utilization of mean and characteristic values of the resistance, the value of  $R_{\theta \Delta} = R_k$  corresponds to the value of resistance calculated with the characteristic values of input random variables,  $X_{i,k}$ , and  $R_{\theta \frac{\Delta}{2}} = R_{\frac{\Delta}{2}}$  value is assessed based on the simulation with intermediate values of random variables according to:

$$X_{i, \frac{\Delta}{2}} = \frac{\mu_{X_i} + X_{i, \Delta}}{2} \quad (5)$$

Finally, the  $\Delta_{\theta}$  represents the distance between the mean value,  $\mu_{\theta}$ , and desired quantile  $F_{\theta}^{-1}(\Phi(-c))$  where  $c$  is a step size parameter ( $c = 1.645$  in this case),  $F_{\theta}^{-1}$  is an inverse cumulative distribution function and  $\Phi$  is the cumulative distribution function of the standardized Gaussian distribution. The following approximation can be used:

$$\Delta_{\theta} = \mu_{\theta} - \mu_{\theta} \cdot \exp \left( -c \cdot \frac{\sqrt{\lambda_1}}{\mu_{\theta}} \right) \quad (6)$$

The global safety factor,  $\gamma_R$ , and the design value of the resistance,  $R_d$ , are assessed based on Eqs. (2–3).

## 3. Case study

A single span railway bridge located in the Czech Republic was analyzed as a case study using semi-probabilistic methods described above. The bridge was put into service 135 years ago with the last major reconstruction before 85 years. A double-track railway line slab bridge spans a local road, supported by two concrete abutments and reinforced concrete bearing seats. Its superstructure comprises 28 cast-in steel I-beams, measuring 11.00 m in width and 8.10 m in length. Inspections of similar structures in the Czech Republic indicate concrete degradation up to 40 mm deep across the beam shell, attributed to deterioration and mechanical damage on the superstructure's bottom. Additionally, the beams' flanges show localized exposure with metal plates suffering from pitting corrosion, reaching depths of up to 3.0 mm.

Non-linear deterministic model of the bridge superstructure was created in the ATENA Science programming shell. The “3D Nonlinear Cementitious 2” material model used to represent the behavior of concrete. For steel, a bilinear stress-strain law without hardening was considered. Rolled I45 steel beams are the main carrying elements of the bridge deck superstructure. The beams are placed in the concrete slab deck. Beams placement was considered on linear supports. The axial distance of the beams in the deck is 390 mm. A typical segment of the bridge deck is shown in Fig. 1 (left). The use of steel beams made of S235 steel and C8/10 concrete class was assumed. The values of the material parameters used for the deterministic analysis correspond to the mean values,  $\mu$ , of these parameters according to the stochastic model of input random variables used for the subsequently performed stochastic analysis; see Tab. 1. The imposed load by rail traffic was considered in accordance with EN 1991-2 (2003), simplified as uniformly distributed over the length of the deck.

### 3.1. Stochastic analysis

Due to the lack of information on the material parameters of concrete and steel used, the values derived from the assumed nominal values typical for the defined strength classes of materials were considered. The stochastic model (Tab. 1) in the form of mean values,  $\mu$ , and coefficients of variation (CoV) of material parameters for concrete was defined based on the value of concrete compressive strength,  $f_c$ , from which other parameters of the concrete were derived according to EN 1992-1-1 (2004) and the fib Model Code 2010 (2013). The probability distribution function (PDF) and material parameters of steel were determined according to Joint Committee on Structural Safety (JCSS, 2001). For the basic stochastic analysis of carrying capacity, 30 random simulations were generated using Latine Hypercube Sampling method and simulated annealing approach to introduce correlations. The mid-span measured load–deflection curves corresponding to simulated realizations of stochastic model are captured in the Fig. 1 (right).

Parameter* [Unit]	$\mu$	CoV	$k^{**}$	$\frac{\Delta}{2} = \frac{\mu + k}{2}$
$E_c$ [GPa]	25.331	0.15	19.601	22.466
$f_t$ [MPa]	1.905	0.30	1.126	1.515
$f_c$ [MPa]	16.0	0.06	8.0	12.0
$G_f [\times 10^{-5} \text{ N/m}]$	12.02	0.128	9.67	10.85
$E_{st}$ [GPa]	210.0	0.03	199.8	204.9
$f_{y,st}$ [MPa]	265.0	0.07	235.0	250.0

\* All the parameters have log-normal (2-par.) PDF

\*\* Characteristic value corresponds to 5 % or 95 % percentile of PDF

Tab. 1: Stochastic model of input random variables.

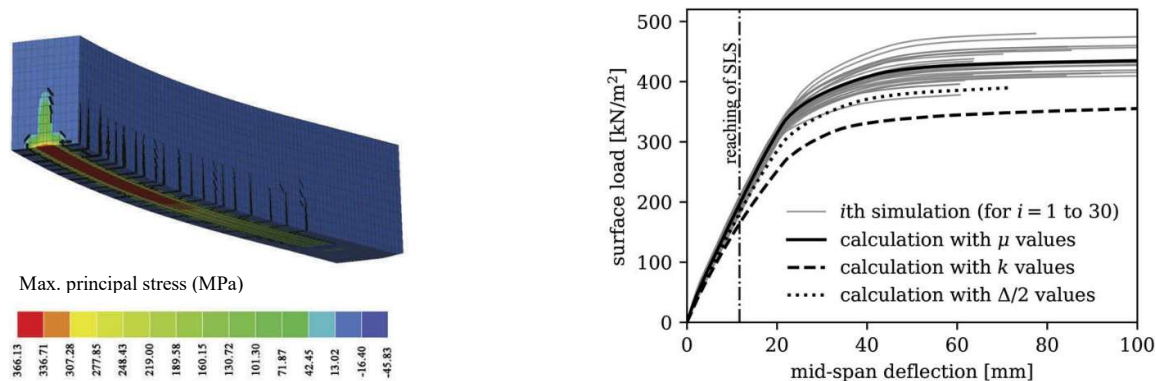


Fig. 1: A typical segment of the bridge deck at ultimate limit state (left), and the load–deflection curves corresponding to simulated realizations of stochastic model (right).

The serviceability limit state (SLS,  $l/600 = 11$  mm) and ultimate limit state (ULS, reaching the yield strength with partial plasticization of the steel beams) were analyzed. The estimation of the design

resistance value at ULS was performed for a reliability index  $\beta = 3.8$  multiplied by a sensitivity coefficient  $\alpha_R = 0.8$  in accordance with EN 1990 (2002) as a quantile corresponding to a failure probability  $p_f = 0.001183$ . When reaching the SLS, i.e. the limit value of the vertical deflection of the typical segment, the design value of the resistance was determined for the reliability index value  $\beta = 0.0$ .

### 3.2. Modelling of degradation processes

In order to take the actual condition of the bridge superstructure into account, modelling of degradation processes was performed. Processes of concrete carbonation due to  $\text{CO}_2$  and consequent following corrosion of cast-in I45 steel beams were modelled using analytical models recommended by the fib Model Code for Service life design (2006). The models take the material parameters as well as the environmental characteristics into account. For detailed description of degradation analysis, see Šomodíková et al. (2022). The stochastic evaluation of analytical degradation was performed on the model. The design values of load carrying capacity for SLS and ULS were then calculated on models with different state of degradation over time using ECoV and eigen ECoV methods (see Chap. 2). The results are summarized in Tab. 2. The results confirmed a gradual decrease in design carrying capacity due to deterioration. Both semi-probabilistic methods give similar results, the ECoV method is slightly more conservative.

	$R_m$	$R_k$	$\frac{R_{\Delta}}{2}$	ECoV design value	Eigen ECoV design value
Load at SLS (0 years)	196.8	164.3	182.1	196.8	196.8
Load at SLS (85 years)	181.8	152.9	168.6	181.8	181.8
Load at SLS (100 years)	178.0	150.8	165.9	178.0	178.0
Load at ULS (0 years)	425.2	342.3	383.6	284.8	292.7
Load at ULS (85 years)	402.4	328.3	365.0	276.3	281.7
Load at ULS (100 years)	395.7	322.1	358.1	270.5	273.6

Tab. 2: Resulting carrying capacity values in  $\text{kN/m}^2$ .

## 4. Conclusions

To accurately simulate structural behavior and current load carrying capacity of existing bridge structure, the study utilized advanced stochastic methods, detailed FEM analysis considering material and geometric non-linearities, and analytical models for deterioration processes. It outlined steps for a numerical deterioration-based probabilistic assessment of structural design resistance.

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